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AN OPTIMAL ALGORITHM FOR SOLVING PROTECTION PROBLEMS IN ELASTIC OPTICAL NETWORKS

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Abstract. Protection in the network is one of the central problems of optical network design and belongs to the NP-hard problem class. The problem becomes increasingly complicated in the Elastic Optical Network (EON), today's new optical network. In the EON network, the protection problem must consider additional factors such as energy consumption, frequency distribution requirements, and distribution of modulation formats. Most previous studies focused on proposing approximate solutions to the problem. This paper presents an exact optimization approach using directed p-cycles and a Column Generation algorithm to solve the protection problem efficiently. Our model integrates energy consumption, frequency slot continuity, and modulation constraints. Experiments on NSFNET and USANET show that the proposed method reduces energy consumption by 10–15% and maintains optimality gaps under 10%, outperforming previous models such as EDPC in both efficiency and scalability.
Keywords: elastic optical network, protection in EON, column generation, p-cycle.

1. Introduction

Each year, the number of telecom network users and connected devices continues to increase. According to Cisco's forecast [1], global Internet users will reach 5.3 billion by 2023. At the same time, users' behavior is shifting toward content- and service-centric demands, such as content delivery networks (CDNs), cloud computing, and video streaming—services primarily hosted in data centers. As these services are highly bandwidth-intensive, these trends have resulted in a sharp rise in overall network traffic, particularly across transport networks [1]. To meet evolving user expectations, network operators are now required to upgrade their infrastructure and enhance traffic engineering capabilities [1].

In order to solve the problem of increasing traffic, elastic optical network (EON) technology was proposed. Unlike traditional Wavelength Division Multiplexing (WDM) networks, EON provides higher levels of elasticity and spectrum efficiency. EONs have an elastic optical spectrum and allocate spectrum in flex-grid slots to meet the demands of connection requests to achieve efficient spectrum consumption [2]. Beyond the difference in resource allocation compared to WDM networks, EONs employ adaptive transmitters that accommodate varying transmission speeds and appropriate modulation formats.

One of the key challenges in optical networks is network protection, which ensures the continuity of network operations in the event of failures such as fiber cuts or flooding at control stations. Such failures can disrupt data transmission [3]. In recent years, a popular protection model has been the p-cycle-based approach. This method utilizes "cyclic" structures, providing short recovery times and efficient sharing of spare capacity.

For EONs, the network protection problem presents even greater challenges. As network traffic continues to grow, optical networking technologies demand higher costs and increased energy consumption. Thus, the first challenge is determining the cost and energy efficiency of the network design to implement protection. The second challenge lies in the complexity of designing protection algorithms for EONs, as the problem incorporates additional constraints such as frequency allocation and modulation format distribution. Previous studies have primarily employed heuristic algorithms to find approximate solutions to the problem [4].

In this paper, we investigate the protection problem in EONs using p-cycles, addressing energy consumption, frequency slot allocation, and modulation format distribution. We propose a mathematical model and employ the Column Generation algorithm to obtain optimal solutions to the problem.

2. Content

2.1. The protection problem in optical networks

Network protection is a recovery method that utilizes pre-allocated protection resources to reroute active traffic from affected channels during failures, thereby ensuring the network's continuity and reliability [1].

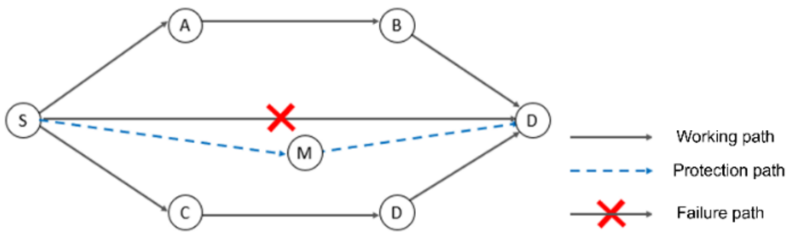


Figure 1. Path Protection in Mesh Networks

2.1.1. The protection problem in WDM networks

The protection problem in WDM networks is described as follows:

Given a graph $G(V, A)$, where V represents the set of nodes and A represents the set of directed links in G. A set of transmission rate levels, denoted as $R = \{10,40,100\}$ Gbps is predefined to provide the required traffic load. The objective is to develop a protection strategy to ensure the network remains operational in the event of link failures within the transmission domain.

Input Data: A set of wavelength channels, λ in the WDM network, and the WDM network topology. The graph $G(V, A)$, where $V = \{v_1, v_2, \dots, v_n\}$ is the set of nodes in the EON network, $A = \{a_1, a_2, \dots, a_m\}$ represents the set of single physical links, with the number of links between v_i and v_j determined by the connectivity degree between v_i and v_j, and A set of demands R, epresenting the number of requests that must be protected. Each request $r \in R$ specifies the traffic load on each link along the route, denoted as l_{vu}.

Output Data: A set of protection paths corresponding to the working paths.

Constraints: There are two constraints: ensuring that all traffic demands are fully protected and ensuring that all single-link failures are fully protected.

Objectives: Optimizing resource utilization in the network and ensuring optimal protection of working paths in the WDM network.

2.1.2. P-cycle protection

P-cycle protection has been regarded as a typical protection solution in recent years due to its use of protective ring structures, which enable short recovery times and high performance in terms of spare capacity utilization [5]. There are several p-cycle protection schemes, including link protection within p-cycles, path protection within p-cycles, and node protection surrounding p-cycles [5]. This method assumes that optical nodes are relatively reliable, so most studies in the literature focus on link and path protection for p-cycles. In this paper, we focus on p-cycles in link protection.

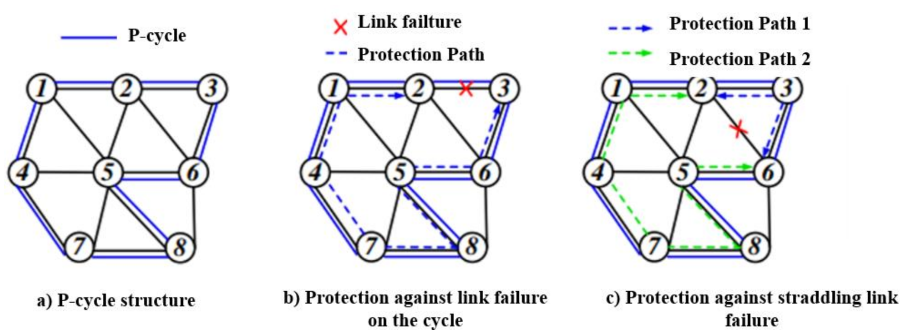


Figure 2. P-cycle protection

Figure 2a presents an 8-node network that incorporates a p-cycle structure, specifically 1-2-3-6-5-8-7-4-1. Within this network, all links can be safeguarded using the p-cycle, categorized as either "on-cycle" links (e.g., links 1-2 or 5-6) or "straddling" links. A "straddling" link connects two nodes that are part of the p-cycle but do not belong to the cycle itself (e.g., links 1-5 or 2-6). Figure 2b demonstrates how protection is applied to "on-cycle" links. When a failure occurs on link 2-3, the end nodes of the failed link redirect traffic through an alternative path, specifically 2-1-4-7-8-5-6-3, along the p-cycle. Figure 2c illustrates the protection mechanism for "straddling" links. In this case, if link 2-6 fails, traffic restoration occurs through two possible protection paths: 2-3-6 and 2-1-4-7-8-5-6, both utilizing the p-cycle. The p-cycle enables pre-configured traffic protection, ensuring that only the two nodes of the failed link need to redirect their traffic along the pre-established paths. Notably, each "on-cycle" link is assigned a single protection path, whereas each "straddling" link benefits from two protection paths.

2.2. The protection problem in EON networks

We propose a solution for finding directed p-cycles to protect EON networks with asymmetric traffic while achieving energy savings. The main objectives of this problem can be summarized as follows:

**** Energy consumption***

e_m^{BVT} in *BVT*: The energy consumption of the BVT for a single FS is given by e_m^{BVT} in Equation (1), which depends on the Transmission Rate (TR) in terms of modulation formats [6].

$$e_m^{BVT} = 1.683 \cdot TR + 91.333 \tag{1}$$

e_v^{OXC} in *BV-OXC*: The energy consumption of a cross-connect node in the bandwidth-variable optical cross-connect (BV-OXC), e_v^{OXC} is expressed in Equation (2). It depends on the fixed base level, additional or reduced levels α, and contributing factors (e.g., power supplies, controllers) [6]. In this context, we consider a switching/grooming degree of 9 at each node and demonstrate that 1 × 9 WSS (Wavelength Selective Switch) consumes 85 W per port [6].

$$e_v^{OXC} = 85 \cdot D_v + 100 \cdot \alpha + 150 \tag{2}$$

e_a^{EDFA} in *EDFA*: The energy consumption of the Erbium-Doped Fiber Amplifier (EDFA) for amplifying signals over a distance of 80 km between each pair of EDFAs depends on the length da of link a. The number of EDFAs required on each link varies according to the link distance. We assume that a single EDFA consumes 100 W, and the energy consumption of EDFAs along link a is calculated using Equation (3) [6].

$$e_a^{EDFA} = [\frac{d_a}{80} + 1].100 \tag{3}$$

**** Frequency Slot (FS) allocation***

The assignment of Frequency Slots (FS) in an optical network follows two key principles:

- *Spectrum Continuity:* In optical networks that lack dynamic spectrum conversion, we assume no spectral conversion occurs. As a result, all links within the same cycle pp must be allocated the same Frequency Slot (FS).

- *Spectrum Contiguity:* FSs allocated to cycles pp are assumed to be adjacent in the spectrum, except for the necessary Guard Band (GB) between them. This approach optimizes spectral resource utilization, allowing p-cycles to share the same FS as long as they do not have overlapping links.

**** Modulation format distribution***

- *Protection Capability:* We consider modulation formats MM, including BPSK, QPSK, 8-QAM, and 16-QAM. The protection capability of an FS is evaluated for each modulation format in MM, corresponding to data rates of 12.5, 25, 37.5, and 50 Gb/s [6].

- *Transmission Range:* The maximum transmission distances for these modulation formats are assumed to be 9600 km, 4800 km, 2400 km, and 1200 km, respectively [7].

❖ The problem of protection in the EON network can be expressed as follows:

Given a graph $G(V, A)$ corresponding to an EON optical network, determine the set of p-cycles to protect the links in the network with asymmetric traffic flow so that the total energy consumption is minimized.

Input data of the problem: The graph G (V, A) is a set $V = \{v_1, v_2, \dots, v_n\}$ being the set of nodes in the EON network. $A = \{a_1, a_2, \dots, a_n\}$ is the set of single links in the network, where each link connects nodes v_i and v_j with the capacity of the link measured by the supported number of sub-channels between v_i and v_j. The set of requirements is the set of demands that need protection; each requirement r ∈ R specifies the traffic load for each link after routing is completed to $\boldsymbol{l_{vu}}$. The set of FS (Frequency Slots) for each link in the network is limited by B, and |B| is the total available FS, and N_G is the number of spectrum guard bands.

Output data of the problem: The set of potential p-cycles capable of protecting the links in the network.

There are three constraints: The maximum spectrum used by the system must not exceed , ensuring the load capacity of the links is protected, and ensuring continuity and spectrum contiguity along the links in the p-cycle.

Objective function: Minimizing the total energy consumption of the p-cycles and minimizing the total number of frequency slots used.

2.3. Optimization model for the protection problem in EON networks

The goal of the problem is to construct a set of p-cycles to protect traffic on the links such that the energy consumption is minimized. The optimization model for the problem is built based on the definition of p-cycle configurations. An EON can be modeled as a graph , where the set of vertices is and the set of directed links is A. Between two adjacent vertices, there are two directed links, for example, the link $v \rightarrow u$ represents a directed link from the node v to node u, which can also be represented by $a \in A$. For readability, we use $\forall i, \forall v, \forall u, \forall m$ and ∀a to represent $\forall i \in I, \forall v \in V, \forall u \in Nv, \forall m \in M$ and $\forall a \in A$. The parameters and variables are declared in Table 1.

Table 1. Network sets and parameters

I	The p-cycle set with the maximum number | I | allowed in EDPC, I_i indicates the i-th p-cycle in I.
N_v	The set of neighboring nodes connected to node v.
M	The set of available modulation levels, where m = 0, 1, 2, and 3 correspond to BPSK, QPSK, 8QAM, and 16QAM, respectively.
d_{vu}	The distance between nodes v and node u in graph G (V, A). The longest link is denoted as L_{max}
TR_m	The bandwidth provided per slot at modulation level m, which is 12.5, 25, 37.5, and 50 Gb/s for BPSK, QPSK, 8-QAM, and 16-QAM, respectively.
e_m^{BVT}	Power consumption of the BVT (Bandwidth Variable Transceiver) at modulation level m
e_v^{OXC}	Power consumption of the BV-OXC (Bandwidth Variable Optical Cross-Connect) at node v
e_a^{EDFA}	Total power consumption of all EDFAs (Erbium-Doped Fiber Amplifiers) on link a

h_m	The maximum transmission range at modulation level m, with values of 9600, 4800, 2400, and 1200 km for BPSK, QPSK, 8-QAM, and 16-QAM, respectively [8]. The maximum and minimum transmission distances are denoted as h_{max} = 9600 km, and h_{min} = 1200 km.
N_G	The guard band width required for one frequency slot (FS).
B	The number of available FSs per fiber link which is 320. |B| is the total number of available frequency slots (FS)
l_{vu}	Traffic load on unidirectional link v → u after routing.
β	A predefined fractional constant satisfying, $\frac{1}{\|V\|} \geq \beta > 0$

Table 2. Variables

$x_{vu}^{i} \in \{0, 1\}$	Equals 1 if link v → u is utilized by I_i, otherwise, it is 0.
$y_{v}^{i} \in \{0, 1\}$	Equals 1 if node v is traversed by I_i,, otherwise, it is 0.
$o_{v}^{i} \in \{0, 1\}$	Equals 1 if node v serves as the root node in I_i, otherwise, it is 0.
$b_{m}^{i} \in \{0, 1\}$	Equals 1 if level m is operated at modulation by I_i, otherwise, it is 0.
$q_{vu}^{i} \in \{0, 1\}$	Equals 1 if link v → u is intended to be protected by I_i, otherwise, it is 0.
$c_{ij} \in \{0, 1\}$	Equals 1 if I_i and I_j have at least one shared link, otherwise, it is 0.
$n_i \in \{0, 32\}$	The number of occupied FSs of I_i is limited to a maximum of 32 because of BVT capacity constraints.
$s_i\ [0, \|B\| - 1]$	The initial index of FSs allocated in I_i.
$o_{ij} \in \{0, 1\}$	Equals 1 if the FSs in I_i start at a lower index than those in I_j; otherwise, it is 0.
$t_b \in [0, \|B\|]$	The highest occupied FS index among all p-cycles
$\pi_{vu}^{im} \in [0, 32]$	The number of FSs assigned by I_i to protect link v → u at modulation level m
$n_{vu}^{i} \in [0, 32]$	The number of FSs used by I_i on link v → u.

**** Potential configurations***

Each configuration is a combination of a p-cycle with a set of links it can protect. For the set of potential configurations in graph G, for each p-cycle i ∈ I and a link $v \rightarrow u$ in the graph, $q_{vu}^{i} \in \{0, 1\}$, where 1 indicates that the link $v \rightarrow u$ is protected by cycle I_i, otherwise it is 0.

**** Variables***

$n_i \in \{0{,}32\}$: The number of frequency slots used by configuration i to provide protection. If not, it is 0.

**** Parameters***

$x_{a}^{i} \in \{0, 1\}$: Determines whether link a is used or not

$b_{m}^{i} \in \{0, 1\}$: Determines whether cycle i is operating with a modulation format m

$q_{a}^{i} \in \{0, 1\}$: Determines whether link a is being protected.

**** Objective Function***

$$\min \theta_1.(E_{BVTs} + E_{OXCs} + E_{EDFAs}) + \theta_1.t_b \qquad (4)$$

The objective is to minimize the total energy consumption of the p-cycles and the number of frequency slots used. Optimization also ensures continuity of the spectrum along the protection paths and enhances the spectrum-sharing capability between the p-cycles. θ_1 and θ_2 are adjustable weighting factors to emphasize the importance of these terms.

The total energy consumption of the network includes:

- E_{BVTs} : *Energy consumption of the BVTs*

$$E_{BVTs} = \sum_{i \in I} \sum_{m \in M} \sum_{a \in A} 2 e_m^{BVT} \pi_m^{BVT} \tag{5}$$

E_{BVTs} is based on the starting and ending nodes of the protection paths. It is the product of the number of frequency slots used and the energy consumption per FS, depending on the modulation format, as specified in Table 2.

- E_{OXCs}*: Energy consumption of the OXCs*

$$E_{OXCs} = \sum_{i \in m} \sum_{v \in V} \sum_{u \in N_v} \frac{n_{vu}^i}{|B|} e_v^{OXC} \tag{6}$$

E_{OXCs} is calculated based on the proportional utilization of the protection path links used on each connection. It depends on the number of FS used relative to the total FS in the fiber.

- E_{EDFAs}: *Energy consumption of the EDFAs*

$$E_{EDFAs} = \sum_{i \in I} \sum_{a \in A} \frac{n_{vu}^i}{|B|} e_a^{EDFA} \tag{7}$$

E_{EDFAs} is calculated similarly to E_{OXCs}, based on the proportional usage of FS

**** Constraints***

Constraints (8) - (10) allocate the order of frequency slots (FS) for each p-cycle.

$$x_a^i + x_a^j - 1 \le c_{ij}, \forall i, j, i \ne j, \forall a \tag{8}$$

$$o_{ij} + o_{ji} = 1, \forall i, j, i \ne j \tag{9}$$

$$s_i + n_i + N_G - s_j \le |B|.(2 - o_{ij} - c_{ij}), \forall i, j, i \ne j \tag{10}$$

Constraint (11): Defines the index t_b, which represents the maximum number of frequency slots used. Minimizing t_b ensures continuity in the spectrum band used for each p-cycle.

$$s_i + n_i \le t_b, \forall i \tag{11}$$

Constraints (12) - (14): Ensure spectrum continuity along the links in a p-cycle.

$$n_a^i \le n_i, \forall i, \forall a \tag{12}$$

$$n_a^i \le x_a^i.32, \forall i, \forall a \tag{13}$$

$$n_a^i \ge n_i - (1 - x_a^i).32, \forall i, \forall a \tag{14}$$

Constraints (15) - (17): Define the protection capability of link a by p-cycle i using modulation format m

$$\pi_a^{im} \le n_i, \forall i, \forall m, \forall a \tag{15}$$

$$\pi_a^{im} \le q_a^i.32, \forall i, \forall m, \forall a \tag{16}$$

$$\pi_a^{im} \le b_m^i.32, \forall i, \forall m, \forall a \tag{17}$$

Constraint (18): Ensures that the maximum traffic capacity of a BVT is 400 Gb/s.

$$\pi_a^{im}.TR_m \le 400, \forall i, \forall m, \forall a \tag{18}$$

Constraint (19): Ensures that all traffic on the links is fully protected.

$$\sum_{i \in I} \sum_{m \in M} \pi_a^{im}.TR_m \ge l_a \forall a \tag{19}$$

2.4. The solution to the protection problem

2.4.1. Solution

One straightforward approach to solving the optimization model in Section 2.2 is to enumerate all possible p-cycle configurations for each connection demand. However, due to the large number of configurations, this method becomes impractical for large input sizes. Fortunately, the problem model can be naturally decomposed, enabling its linear relaxation to be efficiently solved using the Column Generation Algorithm.

The Column Generation Algorithm is a widely used technique for addressing large-scale optimization problems [9]. It works by decomposing the original problem into a Master Problem and one or more Subproblems. The Master Problem consists of simple constraints and aims to find an optimal solution using an initial small set of p-cycle configurations. Meanwhile, the Subproblem, which is also an optimization problem, involves more complex constraints. Its objective is to identify a potential p-cycle configuration that, when added to the Master Problem's configuration set, enhances the objective function. The algorithm continues this process until the Subproblem can no longer identify a viable configuration.

2.4.2. Subproblems

The Subproblem generates potential p-cycle configurations for each protection demand r. The optimization model of the Subproblem is constructed based on the dual values of the constraints in the Master Problem. α^{ij}, β^i, γ_a^i, δ_a^i, μ_a^{im} are the dual values associated with constraints (10), (11), (12), (14), and (15) of the Master Problem. The mathematical model of the Subproblem can be constructed as follows:

**** Objective function***

$$-\sum_{i,j \in I} \alpha^{ij} - \sum_{i \in I} \beta^i + \sum_{i \in I, a \in A} \gamma_a^i - \sum_{i \in I, a \in A} \delta_a^i \cdot q_a \cdot x_a + \sum_{i \in \mathrm{I}, m \in M, \alpha \in A} \mu_a^{im} \cdot b_m \tag{20}$$

The objective function of the Subproblem is constructed using the dual values from the Master Problem to ensure the generation of potential configurations.

**** Constraints***

Constraint (21): Ensures that at most one directed link between two nodes can be used in the directed p-cycle.

$$x_{vu} + x_{uv} \leq 1, \forall i, \forall v, \forall u \tag{21}$$

Constraints (22) and (23): Ensure that if a node v is crossed by a p-cycle, the cycle must have one incoming link and one outgoing link at that node.

$$\sum_{u \in N_v} x_{uv} - \sum_{u \in N_v} x_{vu} = 0, \forall i, \forall v \tag{22}$$

$$y_v = \sum_{u \in N_v} x_{vu}, \forall i, \forall v \tag{23}$$

To ensure the creation of a unique cycle, additional constraints (24) and (25) are introduced to eliminate other conflicting voltage cycles and ensure that the cycle formed is a single connected circuit.

$$f_u - f_v + o_u \geq (1+\beta).x_{vu} - 1, \forall i, \forall v, \forall u \tag{24}$$

$$\sum_{v \in V} o_v \leq 1, \forall i \tag{25}$$

Constraint (26): Ensures the selection of a modulation format compatible with the maximum transmission distance.

$$\frac{\sum_{a \in A} d_a.x_a - q_a.d_a}{h_m} \leq \frac{h_{\max}}{h_{\min}}.(1-b_m) + \frac{L_{\max}}{h_{\min}}.(1-q_a) + b_m, \forall i, \forall m, \forall a \tag{26}$$

Constraint (27): Ensures that only one modulation format can be assigned to a p-cycle.

$$\sum_{m \in M} b_m = 1, \forall i \tag{27}$$

Constraints (28) and (29): Define that the link v $\rightarrow$ u is protected by p-cycle i, provided that the link v $\rightarrow$ u is only a crossing link in the cycle and not a part of the cycle itself.

$$q_{vu} \leq \frac{1}{2}(y_v + y_u), \forall i, \forall v, \forall u \tag{28}$$

$$q_a \leq 1 - x_a, \forall i, \forall a \tag{29}$$

2.5. Column generation algorithm

In the proposed CG-based algorithm, a column refers to a candidate p-cycle used to protect a working link. Since the goal of the Column Generation (CG) method is to reduce the number of variables (columns) in the problem, we first need to formulate the optimization problem with an initial set of feasible columns. Let I_e be the set of all candidate p-cycles that can protect the link $e \in E$ (feasible columns). Next, denote I_e^{cur} as the set of current p-cycles used to protect the link $e \in E$ (i.e., p-cycles that can currently be used to protect the link), and let I be the total set of feasible but not yet available p-cycles for link protection.

Algorithm 1: CG Algorithm for Protection Problem in EON Networks

for e ∈ E **do**

    Initialize the set I_e^{cur}

end for

loop

    Solve the Master problem (MP) using the current set of constraints I_e^{cur} to obtain the optimal solution and dual values.

    for e ∈ E **do**

        Solve the Subproblem (SP) to find a new p-cycle i

        if the reduced cost is negative, then

            Update $I_e^{cur} = I_e^{cur} \cup \{i\}$

        end if

    end for

end loop

Solve the final ILP of the last MP

return Optimal solution

The idea of the CG-based method is presented in Algorithm 1. First, an initial set of columns $\boldsymbol{I^{cur}}$ is created for each link $\boldsymbol{e} \in \boldsymbol{E}$. Then, CG iteratively attempts to find and add new columns that are expected to improve the solution. In each iteration, the Master Problem (MP) is solved. After that, dual values related to the current solution are extracted and used to formulate a subproblem to find a new p-cycle $\boldsymbol{i} \in \boldsymbol{I_e^{can}}, \boldsymbol{e} \in \boldsymbol{E}$, that has a negative reduced cost and can improve the objective function. If such a column is found, it is added to $\boldsymbol{I^{cur}}$. The CG method then proceeds to the next iteration. If no such column is found, the CG algorithm returns the current set $\boldsymbol{I_e^{cur}}, \boldsymbol{e} \in \boldsymbol{E}$, and the obtained solution is considered final.

2.6. Experimental results

In this section, we test and evaluate the proposed solution to find the optimal solution for the protection problem in EON optical networks. The program is written in the Optimization Programming Language (OPL) [10] and uses the CPLEX 12.10.0 library to solve the optimization models for the Master Problem and the Subproblem. The following sections describe the network topology and experimental dataset, followed by an evaluation and comparison of the quality of the proposed solution.

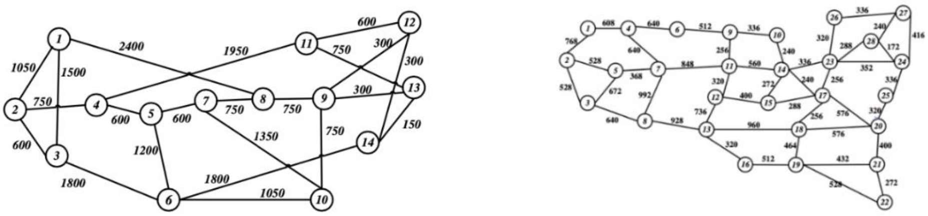
2.6.1. Network and dataset

In this section, the performance of the proposed algorithm is evaluated in terms of energy efficiency and large-scale networks. The network topologies used for simulation are NSFNET (consisting of 14 nodes and 44 directed links) and USANET (consisting of 28 nodes and 90 directed links). Their network topologies are presented in Figure 5.

To assess the effectiveness of the method, we randomly generate bidirectional traffic demand sets for the working paths. We assume that the traffic demand rate follows a uniform distribution within the range of 50–300 Gbps, with an increment of 12.5 Gbps (i.e., 50, 62.5, 75, ..., 287.5, 300 Gbps). The modulation formats used are (BPSK, QPSK, 8-QAM, 16-QAM) based on the transmission distance for each lightpath request.

Here, it is assumed that the data-carrying capacity of each optical fiber link is 4 THz, divided into spectrum slots with a width of 12.5 GHz, allowing for 320 slots per fiber link. The number of protection bands for each request is considered to be one. Connection requests are randomly generated between any source-destination pair among all possible pairs.

All simulations are performed on a server environment running CPLEX 12.06.2 with an Intel(R) Xeon(R) 8-core E5620 CPU and 128GB RAM. First, we evaluate the solution quality of the proposed algorithm. Then, the energy-efficient protection model is compared with the EDPC model proposed in [5].



(a) NSFNET networks *(b) USA networks*

Figure 3. NSFNET and USA networks topology

2.6.2. Experimental evaluation

Table 3 and table 4 summarize the CG results for the NSFNET and USANET network topologies. The Z_{LP} column stores the optimal value of the linear relaxation problem, which also serves as the lower bound of the problem. The Z_{ILP} column stores the optimal solution value of the algorithm. The GAP column represents the optimality gap, calculated using the formula $GAP = \frac{Z_{ILP} - Z_{LP}}{Z_{LP}}$. The next column records the number of columns generated during the algorithm's execution. The CPU column indicates the real execution time of the algorithm in seconds.

Table 3. Experimental results on the NFSNET network

NSFNET network						
Dataset	***Requests***	***Z_{LP}***	***Z_{ILP}***	***GAP (%)***	***Number of columns***	***CPU(s)***
SD1	20	2748.6	2938.7	6.9	45	29
SD2	40	3001.1	3228.7	7.5	90	177
SD3	60	4641.8	5002.7	7.7	122	341
SD4	80	5300.2	5677.4	7.1	168	1089
SD5	100	5903.8	6226.1	5.4	205	1107
SD6	120	7185.4	7532.7	4.8	251	1603
SD7	140	8472.3	8869.1	4.7	303	2309
SD8	160	9793.6	10199.4	4.2	357	3207
SD9	180	11092.8	11478.5	3.5	401	4293
SD10	200	12395.1	12815.6	3.4	453	5701

Table 4. Experimental Results on the USA network

USANET network						
Dataset	***Requests***	Z_{LP}	Z_{ILP}	***GAP (%)***	***Number of columns***	***CPU(s)***
SD1	20	2322.3	2488	7.1	45	62
SD2	40	3613.6	3966.6	9.7	85	460
SD3	60	5340.8	5810.3	8.8	122	1069
SD4	80	7431.6	7942.4	6.8	162	2249
SD5	100	8214.3	8780.2	6.8	200	1891
SD6	120	10065.1	10739.5	6.7	241	2699
SD7	140	11625.3	12421.6	6.4	358	3058
SD8	160	13185.5	14077.6	6.1	472	4436
SD9	180	14745.7	15733.6	5.7	524	5194
SD10	200	16305.9	17389.6	5.4	643	6327

The results in Table 3 and Table 4 show that the CG approach can effectively reduce the size of the ILP problem by reducing the number of columns. Experimental results on NSFNET and USANET show that the algorithm can find the optimal solution for 200 traffic demands in 5701 seconds and 6327 seconds, respectively. For the largest dataset, SD10, the number of columns generated is 453 for NSFNET and 643 for USANET, while the number of possible configurations is very large.

To provide a rigorous performance comparison, our CG method was benchmarked against the energy-efficient protection model presented in [5], which applies an Integer Linear Programming (ILP) approach tailored for flexible grid optical ring metro networks. While [5] focuses on cost and energy trade-offs in a ring topology with distance-adaptive transceivers, our work extends the scope to general mesh networks with asymmetric traffic and p-cycle-based protection.

The results summarized in Tables 3 and 4 demonstrate that the CG approach consistently achieves near-optimal solutions with optimality gaps ranging from 3.4% to 9.7%, and requires reasonable computation times even for large demand sets of up to 200 requests. In contrast, the EDPC model in [5] is generally limited to smaller network sizes due to exponential growth in complexity.

Moreover, our approach yields average energy consumption savings of approximately 10% to 15% compared to the ILP-based solution in [5], primarily due to the efficient frequency slot assignment and modulation format distribution optimized within the CG framework.

In summary, this extended evaluation confirms that the proposed CG-based optimization offers superior energy efficiency and spectral resource utilization for p-cycle protection in EONs compared to existing ILP models such as [5]. This comprehensive comparison highlights the practical benefits of the CG approach for real-world large-scale elastic optical networks.

3. Conclusions

In this paper, we propose an exact and scalable solution for the protection problem in Elastic Optical Networks (EONs) using directed p-cycles. Our optimization model integrates energy consumption, frequency slot allocation, and modulation format adaptation, addressing the complexity of EONs in a realistic and comprehensive manner. To solve the model efficiently, we employed a Column Generation algorithm that enables handling large-scale networks while maintaining near-optimal solutions. Experimental results on NSFNET and USANET demonstrate that our approach reduces energy consumption by 10–15% compared to previous ILP-based models such as EDPC, while achieving optimality gaps below 10% for networks with up to 200 demands. These results confirm the advantages of our method in terms of energy efficiency, scalability, and practical applicability. In future work, we aim to extend the model to dynamic traffic scenarios and investigate its applicability to next-generation optical devices and reconfigurable network architectures.

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