

PRODUCTION OF VECTOR UNPARTICLE PAIRS IN e^+e^- COLLISIONS

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Abstract. We calculate and investigate the production of vector unparticle pairs in e^+e^- collisions for both unpolarized and polarized initial e^+ , e^- beams, and evaluate the dependence of the cross section on the cosine of the scattering angle $\cos\theta$, the energy E_U of the produced vector unparticle, the center-of-mass energy $\sqrt{s}$, the beam polarization coefficients, and the invariant parameters d_U and the energy scale Λ_U of the unparticle physics model. Here, the vector unparticle is treated as an effective massless final-state vector. The results indicate that the differential and total cross sections strongly depend on the considered parameters. The differential cross section reaches its maximum in the forward or backward directions, corresponding to the same or opposite direction relative to the initial electron beams. The total cross section is larger in the low collision-energy region and decreases as the energy of the produced vector unparticle beam increases. The total cross section attains its maximum value when both the e^+ , e^- beams are 100% right-polarized. The cross section becomes significantly larger for smaller values of d_U , while showing less sensitivity to variations in Λ_U within the investigated parameter range. These results provide theory-level estimates that may be useful for exploring constraints on the parameter space of the unparticle physics model within the adopted effective massless-vector approximation.

Keywords: e^+e^- collisions, vector unparticle, cross section, beam polarization, unparticle physics.

1. Introduction

The Standard Model (SM) of particle physics has achieved remarkable success in describing elementary particles and their interactions. In particular, the discovery of the Higgs boson at the Large Hadron Collider (LHC) has provided strong experimental confirmation of the SM. Nevertheless, several fundamental problems remain unresolved, including the nature of dark matter, neutrino masses, the hierarchy problem, and the

absence of gravity in quantum field theory. Therefore, the search for physics beyond the Standard Model (BSM) continues to be one of the primary goals of modern high energy physics.

Among the various BSM scenarios, unparticle physics proposed by Georgi [1] has attracted considerable attention because it introduces a scale-invariant hidden sector. The unparticle model is based on the existence of a scale-invariant hidden sector weakly coupled to SM fields at a high energy scale. At low energies, this hidden sector manifests itself as unparticle operators characterized by non-integer scaling dimensions d_U . Since the pioneering work of Georgi, the phenomenology of scalar, vector, and tensor unparticles has been extensively investigated in many physical processes [2]-[6]. These studies include flavor physics, unparticle physics, and Higgs phenomenology [7], and dark matter phenomenology. In collider physics, unparticles may contribute through both virtual exchange mechanisms and direct production channels, producing observable deviations from SM predictions. In particular, vector unparticles are especially interesting because their interactions may generate characteristic angular distributions and energy dependences in scattering processes. Electron-positron colliders provide an exceptionally clean environment for probing weakly coupled new physics due to their precisely known initial states and relatively small Quantum Chromodynamics (QCD) backgrounds. Over the past decades, many processes e^+e^- colliders, such as fermion-pair production, gauge boson production, Higgs production, monophoton signatures, and exotic invisible channels, have been widely studied in searches for new physics phenomena. Current and future lepton colliders, including the International Linear Collider (ILC), the Circular Electron Positron Collider (CEPC), and the Future Circular Collider in the electron-positron mode (FCC e^+e^-), have received increasing attention because of their extremely high luminosities and precision capabilities [8]-[10]. These facilities are expected to operate in the center-of-mass energy range from several hundred GeV to the multi-TeV region and are considered powerful tools for exploring electroweak precision physics as well as BSM phenomena [11]. Many recent studies have focused on dark sectors, exotic particles, and invisible final states at future e^+e^- colliders [12]. Searches for weakly interacting particles and scale-invariant hidden sectors are among the important topics pursued at future lepton colliders. In the framework of unparticle physics, previous investigations mainly concentrated on unparticle exchange effects in fermion scattering [13], as well as unparticle production processes at other future facilities, such as top quark pair production at photon colliders [14]. In recent years, alternative scenarios involving unparticle physics and extra dimensions, particularly within the framework of the Randall-Sundrum (RS) model, have been extensively investigated to probe signatures of new physics. In this context, previous works have explored various phenomenological aspects of unparticles and other exotic sectors in high energy collisions. Specifically, the production of scalar unparticles in association with photons or Z bosons [15], as well as fermion dark matter production, has been scrutinized in lepton and photon colliders [16]. Furthermore, the significant contributions of unparticles, radions, and gauge bosons to lepton scattering processes such as Møller scattering [17], $e^+e^- \rightarrow e^+e^-$ [18], and $\mu^+\mu^- \rightarrow \mu^+\mu^-$ [19], [20] within the RS framework have been studied. More recently, attention has also been directed toward the production of saxions via vector unparticle

exchange [21], as well as the effects of polarized beams on ZZ boson pair production in unparticle models [22]. In contrast to Ref. [3], which addresses the broader collider phenomenology of unparticle interactions, the present work focuses specifically on the direct production of a pair of vector-unparticle states in e^+e^- collisions through t - and u -channel electron exchange. In addition, the present analysis examines the effects of initial-beam polarization on the differential and total cross sections. Within the effective treatment adopted here, the vector unparticle is represented as a massless effective final-state vector, and conventional two-body kinematics is used. Therefore, the present work provides a process-specific phenomenological analysis rather than a general treatment of the continuum unparticle final state. The process considered is

$$e^+e^- \rightarrow U^\mu U^\mu$$

The vector unparticle is treated as a massless effective final-state vector with $m_U = 0$, so that conventional two-body kinematics gives $E_U = \sqrt{s}/2$ in the center-of-mass frame. We calculate the differential and total cross sections and analyze their dependence on the center-of-mass energy, scattering angle, beam polarizations, scaling dimension d_U , and unparticle interaction parameters. The resulting phenomenological analysis provides a direct assessment of vector-unparticle pair production within the effective description adopted here and may serve as a useful reference for future searches for scale-invariant hidden-sector phenomena at high energy e^+e^- colliders.

2. The process $e^+e^- \rightarrow U^\mu U^\mu$

2.1. Scattering amplitude

The production of a vector unparticle pair in e^+e^- collisions proceeds only via the t - and u -channels through electron exchange, as shown in Figure 1.



Figure 1. The Feynman diagrams describing the process $e^+e^- \rightarrow U^\mu U^\mu$

The effective interaction between the vector-unparticle operator O_U^μ and fermions is given by [3]:

$$\lambda_1 \frac{1}{\Lambda_U^{d_U-1}} \bar{f} \gamma_\mu f O_U^\mu + \lambda_2 \frac{1}{\Lambda_U^{d_U-1}} \bar{f} \gamma_\mu \gamma_5 f O_U^\mu \quad (1)$$

In this work, we assume equal vector and axial-vector couplings, $\lambda_2 = \lambda_1 = \lambda$. Therefore, the interaction is taken to have the V+A chiral structure, and the corresponding fermion–vector-unparticle vertex is

$$i \frac{\lambda}{\Lambda_U^{d_U-1}} \gamma_\mu (1 + \gamma_5) \quad (2)$$

By applying the Feynman rules to Figure 1, we obtain the corresponding scattering amplitude as

$$\begin{aligned}
 M &= \bar{v}(p_2, s_2) i \frac{\lambda}{\Lambda_U^{d_U-1}} \gamma^\mu (1 + \gamma_5) \varepsilon_\mu^*(k_1, \lambda_1) \frac{-i(\hat{q}_u + m_e)}{q_u^2 - m_e^2} \varepsilon_\nu^*(k_2, \lambda_2) i \frac{\lambda}{\Lambda_U^{d_U-1}} \gamma^\nu (1 + \gamma_5) u(p_1, s_1) \\
 &\quad + \bar{v}(p_2, s_2) i \frac{\lambda}{\Lambda_U^{d_U-1}} \gamma^\nu (1 + \gamma_5) \varepsilon_\nu^*(k_2, \lambda_2) \frac{-i(\hat{q}_t + m_e)}{q_t^2 - m_e^2} \varepsilon_\mu^*(k_1, \lambda_1) i \frac{\lambda}{\Lambda_U^{d_U-1}} \gamma^\mu (1 + \gamma_5) u(p_1, s_1) \\
 &= 2i \left(\frac{\lambda}{\Lambda_U^{d_U-1}} \right)^2 \frac{1}{q_u^2 - m_e^2} \varepsilon_\mu^*(k_1, \lambda_1) \varepsilon_\nu^*(k_2, \lambda_2) \bar{v}(p_2, s_2) \gamma^\mu \hat{q}_u \gamma^\nu (1 + \gamma_5) u(p_1, s_1) \\
 &\quad + 2i \left(\frac{\lambda}{\Lambda_U^{d_U-1}} \right)^2 \frac{1}{q_t^2 - m_e^2} \varepsilon_\nu^*(k_2, \lambda_2) \varepsilon_\mu^*(k_1, \lambda_1) \bar{v}(p_2, s_2) \gamma^\nu \hat{q}_t \gamma^\mu (1 + \gamma_5) u(p_1, s_1). \quad (3)
 \end{aligned}$$

The squared scattering amplitude corresponding to Eq. (3), is obtained by summing over the final states using

$$\begin{cases} \sum_{\lambda_1} \varepsilon_{\mu'}(k_1, \lambda_1) \varepsilon_\mu^*(k_1, \lambda_1) = -g_{\mu\mu'} \\ \sum_{\lambda_2} \varepsilon_\nu^*(k_2, \lambda_2) \varepsilon_{\nu'}(k_2, \lambda_2) = -g_{\nu\nu'} \end{cases}$$

and over the initial-state spin configurations using,

$$\begin{aligned} \sum_{s_1} \bar{u}^a(p_1, s_1) u_f(p_1, s_1) &= (\hat{p}_1 + m_e)_f^a \\ \sum_{s_2} \bar{v}^d(p_2, s_2) v_c(p_2, s_2) &= (\hat{p}_2 - m_e)_c^d \end{aligned}$$

The result is

$$\begin{aligned}
 |\overline{M}|^2 &= 128 \left(\frac{\lambda}{\Lambda_U^{d_U-1}} \right)^4 \frac{1}{(q_u^2 - m_e^2)^2} \{ 2(p_1 q_u)(p_2 q_u) - (\mathbf{p}_1 \mathbf{p}_2) \mathbf{q}_u^2 \} \\
 &\quad + 128 \left(\frac{\lambda}{\Lambda_U^{d_U-1}} \right)^4 \frac{1}{(q_t^2 - m_e^2)^2} \{ 2(p_1 q_t)(p_2 q_t) - (\mathbf{p}_1 \mathbf{p}_2) \mathbf{q}_t^2 \} \\
 &\quad + 128 \left(\frac{\lambda}{\Lambda_U^{d_U-1}} \right)^4 \frac{1}{(q_u^2 - m_e^2)(q_t^2 - m_e^2)} \times \{ -(\mathbf{p}_1 \mathbf{p}_2)(q_u q_t) - (p_1 q_u)(p_2 q_t) \}. \quad (4)
 \end{aligned}$$

For polarized e^+, e^- beams

$$\begin{cases} v_{L,R}(p_2, s_2) = \frac{1 \pm \gamma_5}{2} v(p_2, s_2) \\ u_{L,R}(p_1, s_1) = \frac{1 \pm \gamma_5}{2} u(p_1, s_1) \end{cases} \quad (5)$$

we obtain a non-zero scattering amplitude only when both the e^+, e^- beams are right-polarized with $M_{RR} = M$ and $|\overline{M}_{RR}|^2 = |\overline{M}|^2$; whereas for left-polarized beams or opposite polarization configurations, the amplitude vanishes.

2.2. Cross section formula

In the center-of-mass reference frame, we use $p_1^\mu(E_1, \vec{p})$, $p_2^\mu(E_2, -\vec{p})$, $p_3^\mu(E_3, \vec{k})$, $p_4^\mu(E_4, -\vec{k})$, where $E_3 = E_4 = E_U = \frac{\sqrt{s}}{2}$, $E_1 = E_2 = \frac{\sqrt{s}}{2}$, $|\vec{p}| = \sqrt{\frac{s}{4} - m_e^2}$, $|\vec{k}| = \frac{\sqrt{s}}{2}$, and $(p_1 p_2) = \frac{s}{2} - m_e^2$, $q_u^2 = m_e^2 - 2\left(\frac{s}{4} + |\vec{p}||\vec{k}|\cos\theta\right)$, $q_t^2 = m_e^2 - 2\left(\frac{s}{4} - |\vec{p}||\vec{k}|\cos\theta\right)$, $(p_1 q_t) = m_e^2 - \left(\frac{s}{4} - |\vec{p}||\vec{k}|\cos\theta\right)$, $(p_1 q_u) = m_e^2 - \left(\frac{s}{4} + |\vec{p}||\vec{k}|\cos\theta\right)$, $(p_2 q_t) = \frac{s}{2} - m_e^2 - \left(\frac{s}{4} + |\vec{p}||\vec{k}|\cos\theta\right)$, $(p_2 q_u) = \frac{s}{2} - m_e^2 - \left(\frac{s}{4} - |\vec{p}||\vec{k}|\cos\theta\right)$.

Thus, we obtain $|\overline{M}|^2$.

Using these expressions, we calculate the differential cross section (DCS) and the total cross section (TCS) according to the formula:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{k}|}{|\vec{p}|} |M_{fi}|^2 S, \quad (6)$$

where $d\Omega = d\cos\theta d\varphi$, $S = \frac{1}{2!}$, $|M_{fi}|^2 = \frac{1}{(2s_{e^-} + 1)(2s_{e^+} + 1)} |\overline{M}|^2$, $s_{e^-} = s_{e^+} = \frac{1}{2}$.

For polarized e^+ , e^- beams, we have

$$\frac{d\sigma(P_-, P_+)}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{k}|}{|\vec{p}|} \frac{(1 + P_- P_+ + P_- + P_+)}{4} |M_{fiRR}|^2 S, \quad (7)$$

here $|M_{fiRR}|^2 = |\overline{M}_{RR}|^2$.

3. Results and discussion

3.1. Differential cross section of the process $e^+ e^- \rightarrow U^\mu U^\mu$

First, we numerically evaluate and plot the dependence of the DCS on $\cos\theta$ for unpolarized and polarized e^+ , e^- beams. The results are shown in Figure 2. The parameters are chosen as follows: $\sqrt{s} = 3000 \text{ GeV}$ (CLIC), $\lambda = 1$, $\Lambda_U = 10 \text{ TeV}$, $d_U = 1.7$, $P_- P_+ + P_- + P_+ = 0.5$.

From the plots in Figure 2, which describe the dependence of the DCS on $\cos\theta$ for the process $e^+ e^- \rightarrow U^\mu U^\mu$, the characteristic contributions of the interaction channels as well as the influence of the polarization states of the e^+ , e^- beams can be clearly observed. Specifically, for the u -channel (Figure 2a), the DCS reaches a maximum value of about $4.343 \times 10^{-17} \text{ mbarn}$ in the backward scattering direction relative to the incoming

e^+ , e^- beam ($\cos\theta \approx -1$) and attains a minimum value of $1.087 \times 10^{-23} \text{ mbarn}$ in the forward direction of the incoming e^+ , e^- beams ($\cos\theta \approx 1$). As $\cos\theta$ increases from -1 to 1 , the DCS decreases gradually. In contrast, in the t-channel (Figure 2b), the behavior is opposite. In this channel: the DCS increases as $\cos\theta$ varies from -1 to 1 , reaching a maximum value of approximately $4.343 \times 10^{-17} \text{ mbarn}$ at $\cos\theta \approx 1$ and a minimum value of about $1.087 \times 10^{-23} \text{ mbarn}$ at $\cos\theta \approx -1$.

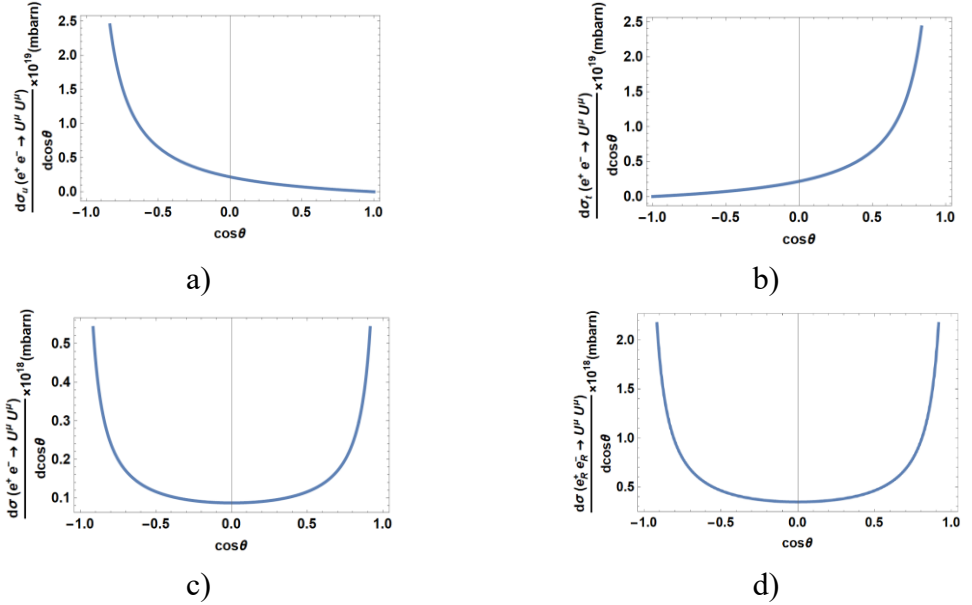


Figure 2. Dependence of the DCS for the process $e^+ e^- \rightarrow U^\mu U^\mu$ on $\cos\theta$

When both the u- and t-channel contributions are included, the DCS is shown in Figure 2c for unpolarized beams and in Figure 2d for polarized beams. From these plots, it can be seen that the DCS reaches its minimum value at the perpendicular scattering angle $\cos\theta = 0$ and its maximum value at $\cos\theta \approx \pm 1$. These values are approximately $8.689 \times 10^{-20} \text{ mbarn}$ and $4.347 \times 10^{-17} \text{ mbarn}$, respectively. In the case of polarized e^+ , e^- beams, the dependence of the DCS on the scattering angle remains unchanged in shape; however, the magnitude of the DCS changes significantly. In this case, the maximum and minimum values are approximately $1.739 \times 10^{-16} \text{ mbarn}$ and $3.476 \times 10^{-19} \text{ mbarn}$, respectively. Both values are larger than those obtained for unpolarized beams.

Next, we evaluate and plot the dependence of the DCS on $\cos\theta$ and E_U for polarized e^+ , e^- beams, with $\sqrt{s} = 2E_U$, $\lambda = 1$, $\Lambda_U = 10 \text{ TeV}$, $d_U = 1.7$, $P_- P_+ + P_- + P_+ = 0.5$.

In Figure 3, we investigate the DCS as a function of both the cosine of the scattering angle, $\cos\theta$ and the energy E_U for polarized e^+ , e^- beams, corresponding to the values $d_U = 1.1, 1.3, 1.5, 1.7, 1.9$. From the plots, it can be seen that for all considered values of d_U , the DCS decreases as $\cos\theta$ increases from -1 to 0 , reaches its minimum value at $\cos\theta = 0$, and continues to decrease as $\cos\theta$ increases from 0 to 1 . At the same time, the

DCS also decreases as E_U increases. When the parameter d_U increases from 1.1 to 1.9, the DCS decreases significantly. For example, at $\cos\theta=0.9$ and $E_U=1500\text{ GeV}$, the DCS takes the values $7.283\times 10^{-9}\text{ mb}$, $4.595\times 10^{-12}\text{ mb}$, $2.899\times 10^{-15}\text{ mb}$, $1.829\times 10^{-18}\text{ mb}$, and $1.154\times 10^{-21}\text{ mb}$, corresponding to $d_U=1.1, 1.3, 1.5, 1.7, 1.9$, respectively.

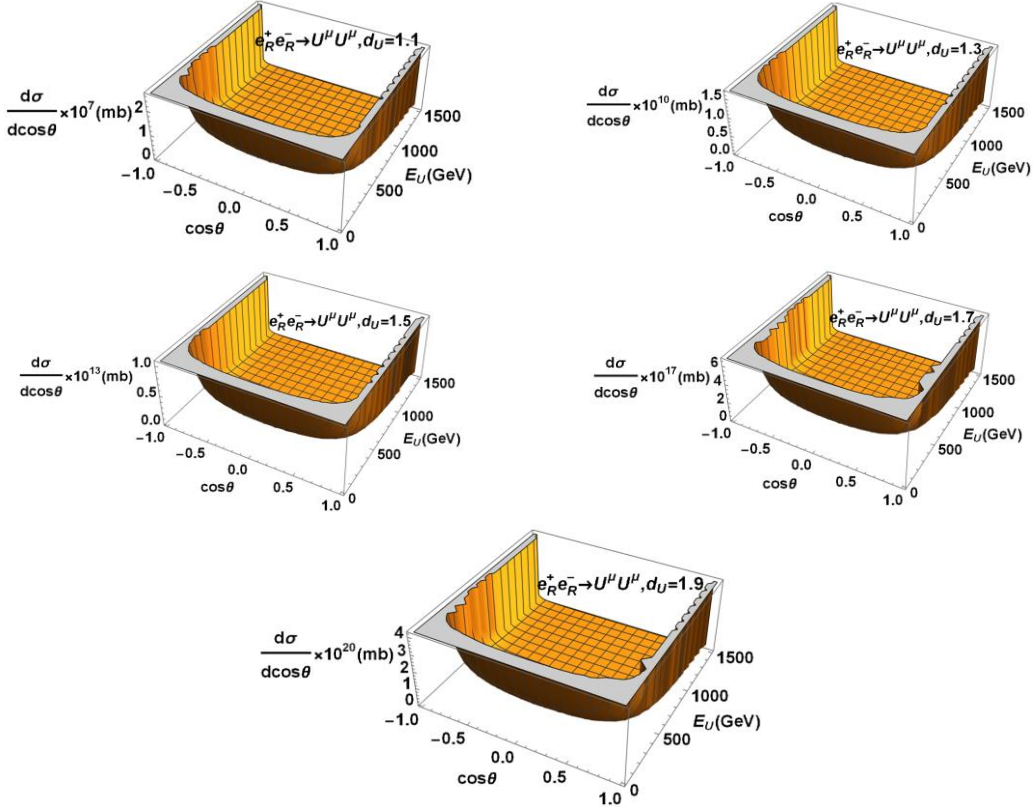


Figure 3. Dependence of the DCS for the process $e^+ e^- \rightarrow U^\mu U^\mu$ on $\cos\theta$ and the energy E_U

3.2. Total cross section of the process $e^+ e^- \rightarrow U^\mu U^\mu$

In the center-of-mass reference frame, from the expression for the squared scattering amplitude, we find that the TCS scales as

$$\sigma \sim \frac{1}{s} \left(\frac{\lambda}{\Lambda_U^{d_U-1}} \right)^4 \quad (8)$$

Therefore, for fixed d_U and Λ_U , the TCS decreases as the center-of-mass energy $\sqrt{s}$ increases. To illustrate this dependence, we evaluate the TCS as a function of $\sqrt{s}$ for unpolarized and polarized e^+ , e^- beams in the t - and u -channels, and their combination. The parameters are chosen as $\lambda=1$, $\Lambda_U=10\text{ TeV}$, $d_U=1.7$, $P_-P_+ + P_- + P_+ = 0.5$ with $\sqrt{s} \leq 3000\text{ GeV}$.

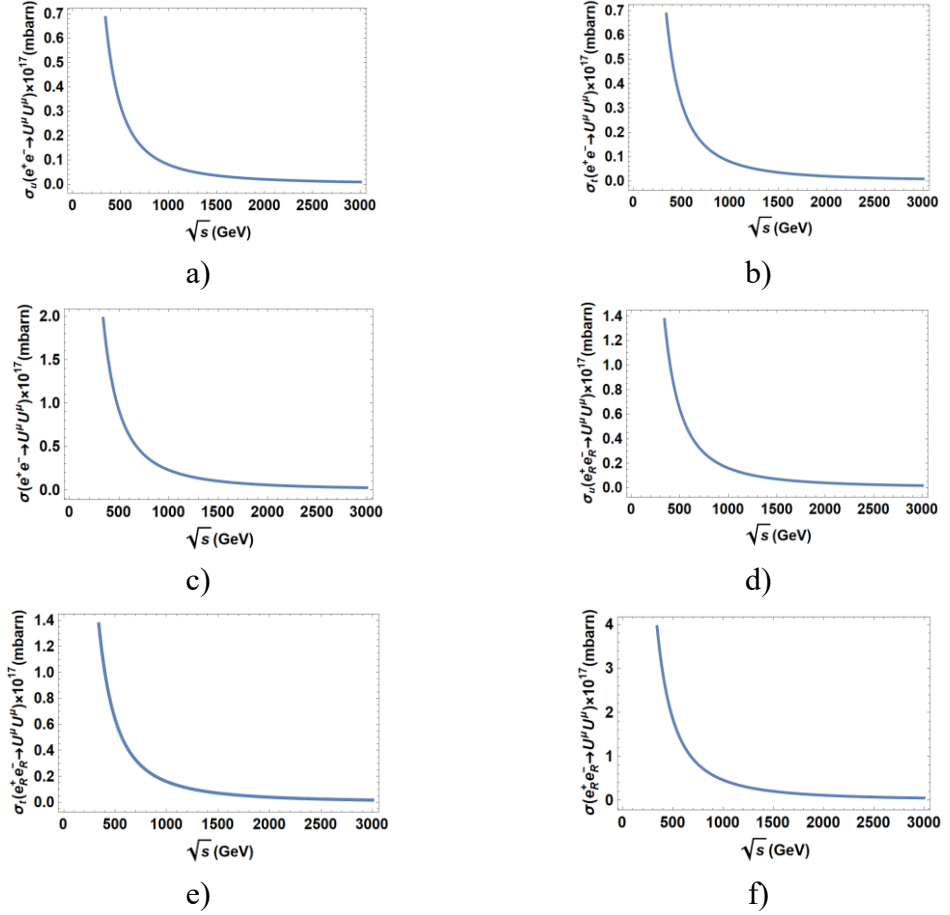


Figure 4. Dependence of the TCS for the process $e^+ e^- \rightarrow U^\mu U^\mu$ on $\sqrt{s}$

From Figure 4, it can be seen that the TCS decreases as $\sqrt{s}$ increases. Specifically, at $\sqrt{s} = 200 \text{ GeV}$ the TCS values for the individual u-channel and t-channel, as well as for both channels combined, are $1.999 \times 10^{-17} \text{ mbarn}$, $1.997 \times 10^{-17} \text{ mbarn}$, and $5.757 \times 10^{-17} \text{ mbarn}$ respectively when e^+, e^- beams are unpolarized. These values are $7.994 \times 10^{-17} \text{ mb}$, $7.994 \times 10^{-17} \text{ mb}$, and $2.303 \times 10^{-16} \text{ mb}$ respectively when the e^+, e^- beams are both right-polarized. Meanwhile, at $\sqrt{s} = 3000 \text{ GeV}$, the TCS values for the individual u and t channels, and for both channels combined, are $8.883 \times 10^{-20} \text{ mbarn}$, $8.883 \times 10^{-20} \text{ mbarn}$, and $2.557 \times 10^{-19} \text{ mbarn}$ respectively when the e^+, e^- beams are unpolarized. These values are $3.553 \times 10^{-19} \text{ mb}$, $3.553 \times 10^{-19} \text{ mb}$, and $1.023 \times 10^{-18} \text{ mb}$ respectively when both the e^+ and e^- beams are right-polarized.

For illustration, we consider representative high energy $e^+ e^-$ collider scenarios. The 500 GeV benchmark is motivated by the physics programme of the International Linear Collider (ILC) [8], [11], while the multi-TeV benchmarks are representative of the energy range considered for future linear colliders such as the ILC and CLIC [8], [11], [23]. The integrated luminosities adopted in Tables 1 and 2 are phenomenological assumptions used

to estimate the corresponding sensitivity to the unparticle scale Λ_U , rather than exact projections of a specific collider running programme. The sensitivity to Λ_U as a function of the scaling dimension d_U is evaluated using the standard event-yield criterion $N = \sigma L_{\text{int}}$, with a minimum event requirement of $N_{\text{min}} = 10$. This approach follows the general strategy adopted in previous collider studies of unparticle phenomenology [24].

Table 1. Threshold cross section σ and projected lower limits on Λ_U for various scaling dimensions d_U at $\sqrt{s} = 500\text{GeV}$, assuming an integrated luminosity of $L = 10^3 \text{fb}^{-1}$ and a minimum event requirement of $N_{\text{min}} = 10$.

d_U	Threshold cross section $\sigma(\text{mb})$	$\Lambda_U(\text{GeV})$
1.1	10^{-14}	2.575×10^{20}
1.3	10^{-14}	6.362×10^6
1.5	10^{-14}	1.208×10^4
1.7	10^{-14}	8.238×10^2
1.9	10^{-14}	185.295

Table 2. Threshold cross section σ and projected lower limits on Λ_U for various scaling dimensions d_U at $\sqrt{s} = 1000\text{GeV}$ and 3000GeV , with an upgraded integrated luminosity of $L = 8 \times 10^3 \text{fb}^{-1}$ and $N_{\text{min}} = 10$

d_U	Threshold cross section $\sigma(\text{mb})$	$\Lambda_U(\text{GeV}), \sqrt{s} = 1000\text{GeV}$	$\Lambda_U(\text{GeV}), \sqrt{s} = 3000\text{GeV}$
1.1	1.25×10^{-15}	1.457×10^{21}	5.994×10^{18}
1.3	1.25×10^{-15}	1.134×10^7	1.817×10^6
1.5	1.25×10^{-15}	1.709×10^4	5.696×10^3
1.7	1.25×10^{-15}	1.055×10^3	481.43
1.9	1.25×10^{-15}	224.638	122.016

The projected sensitivity limits on Λ_U as a function of the scaling dimension d_U are evaluated for representative operating energies of future high energy e^+e^- linear colliders, including the ILC and CLIC. The luminosities adopted in Tables 1 and 2 are phenomenological benchmark assumptions used to illustrate the sensitivity of the process to the unparticle scale. All calculations adopt a conservative minimum event requirement of $N_{\text{min}} = 10$. Specifically, Table 1 presents the projected limits for $\sqrt{s} = 500\text{GeV}$ with an integrated luminosity of $L = 10^3 \text{fb}^{-1}$, while Table 2 summarizes the results for $\sqrt{s} = 1000$ and 3000GeV using an illustrative upgraded luminosity of $L = 8 \times 10^3 \text{fb}^{-1}$.

Across all scenarios, the projected lower limit on Λ_U decreases rapidly with increasing d_U . Within the effective description adopted here, the validity of the contact-interaction approximation requires the characteristic interaction scale to satisfy $\Lambda_U > \sqrt{s}$. As indicated by Tables 1 and 2, this condition is not satisfied for $d_U \geq 1.9$ at $\sqrt{s} = 500 \text{ GeV}$ and 1000 GeV , and for $d_U \geq 1.7$ at 3000 GeV , because the corresponding projected values of Λ_U fall below the center-of-mass energy. These parameter regions should therefore be regarded as lying outside the self-consistent validity range of the effective description adopted here.



Figure 5. Dependence of the TCS for the process $e^+ e^- \rightarrow U^\mu U^\mu$ on the polarization coefficients P_+ and P_- of the e^+ and e^- beams for $d_U = 1.7$.

In Figure 5, we evaluate the polarization dependence based on the polarization coefficients P_- and P_+ of the e^- and e^+ beams, respectively. The degree of polarization for each beam varies within the interval $[-1, 1]$ (i.e., -100% to 100%), where positive and negative values correspond to right-polarized and left-polarized beams, respectively. In this case, the TCS depends on these coefficients through the factor $(1 + P_- P_+ + P_- + P_+)$. As shown in Figure 5, the TCS reaches its maximum when $P_- = P_+ = 1$ (or 100%), where both e^- and e^+ beams are right-polarized. For the experimental scenario at the ILC, with $P_- = 80\%$ and $P_+ = 30\%$, the TCS at $\sqrt{s} = 3000 \text{ GeV}$ is $4.767 \times 10^{-9} \text{ mb}$ for $d_U = 1.1$ and $7.555 \times 10^{-22} \text{ mb}$ for $d_U = 1.9$. At the CLIC, with $P_- = 80\%$ and $P_+ = 0\%$, the corresponding TCS values are $3.667 \times 10^{-9} \text{ mb}$ for $d_U = 1.1$ and $5.812 \times 10^{-22} \text{ mb}$ for $d_U = 1.9$. For unpolarized e^- and e^+ beams, $P_- = P_+ = 0$, the TCS values are $2.037 \times 10^{-9} \text{ mb}$ for $d_U = 1.1$ and $3.229 \times 10^{-22} \text{ mb}$ for $d_U = 1.9$. Thus, at $\sqrt{s} = 3000 \text{ GeV}$, when the e^- and e^+ beams are polarized, the TCS is enhanced by factors of approximately 2.34 for the ILC configuration and 1.8 for the CLIC configuration, compared with the unpolarized case, for both $d_U = 1.1$ and $d_U = 1.9$.

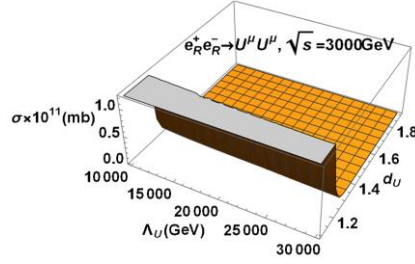


Figure 6. Dependence of the TCS for the process $e^+e^- \rightarrow U^\mu U^\mu$ on Λ_U and d_U for $P_- = P_+ = 0$

In Figure 6, we investigate the TCS with respect to the parameters Λ_U and d_U of the unparticle physics model. Here, Λ_U takes values ranging from 10 *TeV* to 30 *TeV*, and d_U takes values from 1.1 to 1.9. From the graph, it can be seen that the TCS yields larger values in the small d_U region, and it decreases very rapidly as d_U increases from 1.1 to 1.3. For d_U values increasing from 1.3 to 1.9, the total cross section decreases gradually. The dependence of the TCS on the parameter Λ_U is also illustrated in the figure. When d_U is fixed, we observe that the TCS changes slowly as Λ_U takes values from 10 *TeV* to 30 *TeV*. Specifically, when fixing $d_U = 1.1$ the TCS reaches values of $2.037 \times 10^{-9} \text{ mbarn}$, $1.544 \times 10^{-9} \text{ mbarn}$, and $1.313 \times 10^{-9} \text{ mbarn}$ corresponding to the parameter Λ_U values of 10 *TeV*, 20 *TeV*, and 30 *TeV*, respectively.

4. Conclusions

In this work, the vector unparticle is treated as a massless effective final-state vector, and therefore the results presented here do not constitute a general treatment of unparticle physics. We have investigated the differential and total cross sections for the direct production of vector-unparticle pairs in e^+e^- collisions through the t - and u -channel electron exchange diagrams. The DCS shows a clear dependence on the scattering angle and the energy E_U of the produced vector unparticle. The largest DCS is obtained when the direction of the produced vector-unparticle beam is the same as or opposite to the direction of the initial electron beam, corresponding to $\cos \theta \approx \pm 1$, while the smallest values occur at $\cos \theta = 0$. The DCS also decreases as E_U increases. The TCS decreases with increasing center-of-mass energy $\sqrt{s}$. Thus, the TCS is larger in the lower center-of-mass energy region considered in this work.

The polarization of the initial e^+ and e^- beams has a significant effect on the cross section. The cross section is non-zero only when both the e^+ and e^- beams are right-polarized, and it reaches its maximum when both beams are 100% right-polarized. At $\sqrt{s} = 3000 \text{ GeV}$, for the polarization configuration considered for the ILC, $P_- = 80\%$ and $P_+ = 30\%$, the TCS is $4.767 \times 10^{-9} \text{ mb}$ for $d_U = 1.1$ and $7.555 \times 10^{-22} \text{ mb}$ for $d_U = 1.9$. For

the CLIC configuration with $P_- = 80\%$ and $P_+ = 0\%$, the corresponding values are $3.667 \times 10^{-9} mb$ and $5.812 \times 10^{-22} mb$, respectively. These TCS values are higher than those obtained for unpolarized e^+ and e^- beams.

The cross section is strongly dependent on the unparticle scaling dimension d_U . It is larger for smaller d_U and decreases rapidly as d_U increases, particularly in the low- d_U region. The dependence on Λ_U is weaker, with the cross section decreasing gradually as Λ_U increases from 10 to 30 TeV . For the sensitivity estimates with $N = \sigma L_{int}$ and $N_{min} = 10$, the projected lower limit on Λ_U decreases as d_U increases. The effective description requires $\Lambda_U > \sqrt{s}$. Therefore, the regions with $d_U \geq 1.9$ at $\sqrt{s} = 500$ and 1000 GeV , and $d_U \geq 1.7$ at $\sqrt{s} = 3000 GeV$, are outside the self-consistent validity range because the projected lower limit on Λ_U falls below $\sqrt{s}$.

Note for contributor:

- Short bio: Dao Thi Le Thuy is a lecturer, and Le Nhu Thuc is a staff member at Hanoi National University of Education, Vietnam. Their research interests include theoretical particle physics, quantum field theory, collider phenomenology, and Beyond Standard Model physics.

- Author's contributions: Dao Thi Le Thuy: Conceptualization, methodology, formal analysis, numerical calculations, visualization, writing - original draft. Le Nhu Thuc: Conceptualization, supervision, validation, writing - review and editing. Both authors read and approved the final manuscript.

Conflict of interest. The authors declare that they have no conflict of interest.

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