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BUILDING A RUBRIC FOR ASSESSING MATHEMATICAL COMMUNICATION COMPETENCE IN GRADE 5 STUDENTS

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Abstract. Mathematical communication competence is a core component of mathematical competence, helping students to receive, construct, and express mathematical thinking effectively and accurately. However, in reality, primary school teachers still have difficulty in assessing students' mathematical communication competence, including 5th graders, leading to an ineffective impact on the teaching process to develop students' mathematical communication competence. Based on the understanding of theories of mathematical communication competence and the requirements presented in the 2018 National Mathematics Education Curriculum, this article presents a rubric for assessing mathematical communication competence in grade 5 students. This assessment tool is designed to help teachers accurately assess the mathematical communication competence of grade 5 students.
Keywords: assessment, mathematical communication competence, grade 5 students.

1. Introduction

The 2018 General Education Curriculum has shifted its focus from imparting knowledge to developing students' qualities and abilities, emphasizing the philosophy of assessment for the progress of learners. According to this orientation, assessment is not only aimed at determining learning outcomes but also aims to support the learning process and develop comprehensive abilities for students. To achieve that goal, teachers need to use a variety of assessment methods and tools to fully and accurately identify the level of achievement of students' abilities, including mathematical communication ability.

According to the National Council of Teachers of Mathematics (NCTM) in the document *"Principles and Standards for School Mathematics"* (2000), mathematical communication is an important means for students to share mathematical ideas, clarify understanding, and develop thinking through dialogue and discussion. Notably, NCTM emphasizes that communicating mathematical ideas does not stop at describing or summarizing the process but needs to include clear and accurate mathematical reasoning. NCTM also highlights the role of dialogue and interaction, where students participate in discussions, listen, and debate opinions to develop a deeper understanding of mathematics [1].

In addition to NCTM, many other studies have clarified aspects of mathematical communication competence. Malim Muhammad, Joko Purwanto, and Octarina Christiyansyah (2021) also expanded the notion of mathematical communication to include not only speech and text but also graphical representations such as diagrams, tables, and other visual forms [2].

In Vietnam, mathematical communication competence is one of the five components of mathematical competence specified in the 2018 National Mathematics Education Curriculum issued by the Ministry of Education and Training. This competence helps students effectively use mathematical language, common language, and non-verbal expressions (body movements, facial expressions, gestures, etc.) to receive, express, and demonstrate mathematical thinking accurately and clearly. Moreover, mathematical communication competence plays a fundamental role in supporting the development of other mathematical competence components such as mathematical thinking and reasoning, mathematical problem-solving, and the ability to use mathematical tools and means [3].

For primary students, 5th grade marks the transition from the foundation level to secondary school, where the requirements for thinking, expression, and mathematical communication become more complex and diverse. At this point, students have generally formed basic learning skills, logical thinking, and initial communication competence in the context of mathematics. Therefore, assessing the mathematical communication competence of 5th graders not only helps to review the level of achievement of the required competence but also serves as a basis for preparing a solid foundation for the next stage of learning. Simultaneously, this assessment process contributes to the early detection of limitations in students' competence in expressing, presenting, and explaining mathematical problems, thereby allowing teachers to adjust teaching activities appropriately.

In Vietnam, there is currently a lack of research dedicated to building a rubric for assessing mathematical communication competence in grade 5 students, particularly within the context of the 2018 National Mathematics Education Curriculum. This study aims to address this gap by developing a practical assessment tool to support the evaluation and enhancement of mathematical communication competence in primary education. Within the framework of this article, we first systematize key concepts related to mathematical communication competence and describe specific indicators at various proficiency levels. Based on this foundation, we propose a rubric for assessing mathematical communication competence in grade 5 students and apply it to assess the competence of students in Class 5A4 at Chu Van An Primary School, Tay Ho district, Hanoi, during the middle of the second semester of the 2024 - 2025 academic year.

2. Content

2.1. Mathematical communication competence

The term "mathematical communication" first appeared in mathematics education research in the late twentieth century, when educators began to recognize the important role of communication in the acquisition and expression of mathematical knowledge. Isoda (2008) argues that mathematical communication is not simply the use of language but extends to various forms, including gestures, real or virtual models, drawings, writings, graphs, tables, and other devices [4]. Sunardi and Prastiti (2023) also agree with Isoda's view, emphasizing that mathematical communication competence involves communicating and explaining mathematical ideas through different media such as drawings, tables, diagrams, formulas, and demonstrations [5].

Agustina, Nurdin, Ansari, and Sukriyah (2023) define mathematical communication competence as the ability to convey ideas or information from one person to another through certain means, while providing logical reasoning for statements, converting ordinary verbal

language into mathematical language, and illustrating mathematical ideas with appropriate descriptions [6].

In Vietnam, Vu Thi Binh (2016) defines mathematical communication competence as the ability to understand mathematical problems through verbal, written, and graphical communication; and to effectively use mathematical language combined with natural language to exchange, explain, argue, and clarify mathematical ideas accurately and logically within specific contexts [7].

Thus, mathematical communication competence is the ability of students to understand and use mathematical language, common language, and physical movements, while confidently presenting and expressing mathematical ideas during interactions with others.

According to the 2018 National Mathematics Education Curriculum [3], the following manifestations of mathematical communication competence are expected of primary students:

- *Requirement 1*: Demonstrate comprehension by listening to, reading, and taking notes of (summarizing) key mathematical information presented in the text or conveyed by others (at a basic level), thereby identifying the problem that needs to be solved.

- *Requirement 2*: Present and express (speak or write) mathematical content, ideas, and solutions in interaction with others (not yet required to express fully and accurately). Raise and answer questions during argumentation and problem-solving.

- *Requirement 3*: Use mathematical language combined with common language and physical gestures to express mathematical content in simple situations.

- *Requirement 4:* Demonstrate confidence when answering questions, presenting, and discussing mathematical content in simple situations.

Thus, synthesizing the perspectives of the aforementioned scholars along with the descriptors of mathematical communication competence outlined in the 2018's National Mathematics Education Curriculum, the research team defines mathematical communication competence as the ability of students to comprehend and subsequently use mathematical language, everyday language, and physical gestures, while demonstrating confidence in presenting and expressing mathematical ideas and content in interaction with others.

2.2. The levels of mathematical communication competence

According to Ross Turner, Werner Blum, and Mogens Ni [8], there are four levels of mathematical communication competence, described as follows:

- *Level 1:* Understand short sentences or phrases relating to concepts that give immediate access to the context, where all information is directly relevant to the task, and where the order of information matches the steps of thought required to understand what the task requests. Constructive communication at this level involves only the presentation of a single word or numeric result.

- *Level 2:* Identify and link relevant elements of the information provided in the text and other related representations, where the material presented is more complex or extensive than short sentences and phrases, or where some extraneous information may be present. Any constructive communication required is simple; for example, it may involve writing a short statement or calculation, or expressing an interval or a range of values.

- *Level 3:* Identify and select elements to be linked, where repeated cycling within the material presented is needed to understand the task; or understand multiple elements of the context or task and their links. Any constructive communication at this level involves providing a brief description or explanation, or presenting a sequence of calculation steps.

- *Level 4:* Identify, select, and understand multiple context or task elements and the links between them, involving logically complex relations (such as conditional or nested statements).

Any constructive communication would involve presenting argumentation that links multiple elements of the problem or solution.

The levels proposed by Turner et al. delineate the proficiency levels of mathematical communication competence by distinguishing between the stages of receiving and expressing mathematical content and ideas. The research team considers this theoretical framework to be appropriate for building a rubric for assessing mathematical communication competence in Grade 5 students in Vietnam.

2.3. Rubric for assessing mathematical communication competence in grade 5 students

Based on theoretical research regarding the definition of mathematical communication competence, the manifestations of mathematical communication competence for primary students stated in the 2018 National Mathematics Education Curriculum, and the theory of four levels of mathematical communication competence proposed by Ross Turner, Werner Blum, and Mogens Niss, the research team developed a rubric for assessing the mathematical communication competence in grade 5 students as follows:

Table 1. Rubric for assessing mathematical communication competence in grade 5 students

Requirement	Level 1	Level 2	Level 3	Level 4
Requirement 1: Demonstrate comprehension by listening to, reading, and taking notes of (summarizing) key mathematical information presented in the text or conveyed by others (at a basic level), thereby identifying the problem that needs to be solved.	Students understand short sentences or simple phrases related to mathematical content when listening or reading, in which the information is presented clearly in an order that corresponds to the steps of thinking, directly related to the task. Students note down some basic keywords and recognize the problem to be solved in a simple situation.	Students identify and link important mathematical information elements in more complex written or spoken text (containing multiple sentences or some information that is not directly related). Students write a short sentence or simple summary of the problem to be solved.	Students identify and select important mathematical elements to be linked in more complex written or spoken texts, requiring an understanding of the relationships between information. Students summarize content in a short paragraph and recognize problems to be solved when elements are not presented in a clear sequence.	Students identify, select, and understand multiple elements of a task and analyze logical relationships in written or spoken text, including conditional or embedded statements. Students summarize information in a logical, systematic manner and recognize mathematical ideas even when they are embedded in complex information.
Requirement 2: Present and express (speak or write) mathematical content, ideas, and solutions in interaction with others (not yet required to express fully and	Students can say or write a word or number related to a problem in interaction with others, but cannot express their ideas or thought processes.	Students can speak or write a short sentence or calculation to describe a mathematical idea or solution in interaction with others. When reasoning, students can ask	Students can express a short paragraph or a sequence of computational steps to describe a problem-solving approach in interaction with others. When reasoning,	Students can present a coherent argument, linking many elements of the problem to explain it logically and coherently. When arguing, students can ask and answer higher-level deductive questions

accurately). Raise and answer questions when arguing and solving problems.		simple questions related to the problem and answer them in a short sentence.	students can ask and answer questions to clarify issues and provide more reasonable explanations.	and explain the reasons for choosing the solution method more clearly.
Requirement 3: Use mathematical language in combination with common language and physical gestures to express mathematical content in simple situations.	Students can use a word, a numerical result, or a simple mathematical symbol to express mathematical content. The use of mathematical language is limited, mainly at the level of recognizing and reproducing basic information without a complex combination of mathematical language and common language.	The student can use a short sentence that combines mathematical and everyday language, but it is still simple and not coherent. The student can use pointing, drawing diagrams, or simple illustrations to support the expression, but cannot fully explain the reasons.	Students can use both mathematical and everyday language to describe a problem or explain a concept more clearly. In their presentations, students combine drawing diagrams, charts, or using their hands, and demonstrate a series of linked steps.	Students can connect many elements of the problem to reason and present coherent arguments by flexibly using mathematical language combined with common language and physical movements.
Requirement 4: Demonstrate confidence when answering questions, presenting, and discussing mathematical content in simple situations.	Students can answer math questions with one word and one number. Communication is still passive, mainly short responses to questions; there is no active discussion, and they still speak softly and hesitantly.	The student can speak in short sentences or illustrate a calculation. Speech is clearer but still hesitant, and interacts with others using simple body movements.	Students can express their thoughts clearly, combining mathematical language and common language to explain. Students can ask questions or respond to friends' opinions in discussions, showing initiative in mathematical communication. Confidence is shown through a clear voice, eyes directed towards the listener.	Students present ideas logically and convincingly, combining mathematical language, common language, and visual illustrations accurately. Students actively discuss, ask questions, argue, and reasonably defend their views. Confidence is demonstrated through a decisive voice, appropriate use of body language, knowing how to adjust speaking speed, and creating attraction for listeners.

2.4. Applying a rubric for assessing mathematical communication competence in grade 5 students

Based on the theoretical foundations presented, particularly the rubric for assessing mathematical communication competence in Grade 5 students, the research team employed this rubric to evaluate the mathematical communication competence of students in Class 5A4 at Chu Van An Primary School, Tay Ho District, Hanoi City, with a total of 44 students participating.

Test - Assessment: mathematical communication competence (*Time allowed: 35 minutes)*

Exercise 1 (Assessing requirement 1 and requirement 2 - Paper-based): Read the following problem, summarize the content for each specific item, then discuss and provide a written solution for each item.

Primary School A organized a field trip for 5th graders to the Thien Duong Bao Son tourist area. A total of 120 students participated, divided equally among 4 buses.

(a) How many students were there in each bus?

(b) Each bus consumed 15 liters of gasoline for the entire trip. The price of gasoline was 25.000 VND per liter. How much did the school have to pay for gasoline for the trip?

(c) Each student contributed 50.000 VND for the trip. How much money did the school have left after paying for gasoline?

(d) The school plans to organize the trip again in the summer, but the price of gasoline has increased by 10%. Find the difference in the amount the school would have to pay for gasoline between the summer and other seasons

Exercise 2 (Assessing requirement 3 - Oral presentation): Read the following problem, discuss, and present the solution on the board.

Problem: There are 3 candy packages with different masses. The first candy package weighs $\frac{1}{4}$ kg. The second candy package weighs as much as the first candy package plus half the third candy package. The third candy package weighs twice the first candy package plus the second package. Calculate the mass of each candy package.

Exercise 3 (Assessing requirement 4 - Oral presentation): Read the following problem, discuss, and present the solution on the board.

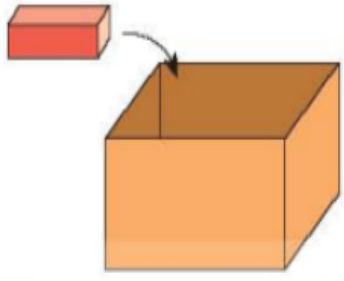

Figure 1. a visual representation of the math problem

Problem: Nam wants to arrange rectangular wooden blocks, each with dimensions 4 cm in length, 1 cm in width, and 1 cm in height, into a rectangular box with dimensions 8 cm in length, 4 cm in width, and 4 cm in height. What is the maximum number of wooden blocks that Nam can arrange inside the box?

Through students' answers, teachers can assess students' mathematical communication competence, thereby adjusting and supporting students to improve or develop corresponding expressions. We take the example of student Le Hai Chau:

- Level 1 (Item a): The student understood and recorded the key information regarding the number of students and the number of cars, and correctly identified that the task was to find the number of students per car.

- Level 2 (Item b): The student linked the initial information with the new information about the number of liters of gasoline consumed by each vehicle and the total number of vehicles, and used it to calculate the total gasoline cost. The student provided a summarized response.

- Level 3 (Item c): The student connected the information about the amount of money contributed by one student with the total number of students, and combined it with the result from point b (the total gasoline cost) to calculate the amount of money remaining after expenses.

- Level 4 (Item d): Although the question did not directly request the gasoline cost for the summer, the student identified that the primary task was to calculate the gasoline cost after a 10% price increase. The student recalculated the new gasoline price, referred back to point b, and performed the appropriate calculations to find the difference between the summer cost and the original cost.

According to the rubric for assessing the mathematical communication competence in grade 5 students, student Le Hai Chau achieved level 4 for requirement 1**.**

Through the student's work, it is evident that the mathematical ideas are expressed mainly in the form of short, discrete sentences rather than as a complete, logical sequence of calculation steps, particularly in Exercise 2, item d:

- Item a**:** "120 students are divided equally among 4 buses, so each bus has 30 students".

- Item b**:** "Each car uses 15 liters; there are 4 cars, so 15 × 4 × 25,000 (VND) = 1,500,000 VND".

- Item c**:** "Each person pays 50,000 VND; the total is 120 × 50,000 VND. After deducting the gasoline cost, 4,500,000 VND remains".

- Item d**:** "The price of gasoline increased by 10%, so the new price is 27,500 VND per liter. Calculate again using the new price".

Compared with the rubric criteria for requirement 2, student Le Hai Chau demonstrated performance corresponding to Level 2**.**

For Exercise 3, the research team organized a group discussion and invited student Le Hai Chau to present the solution on the board. The team observed and assessed both the discussion process and the student's presentation, focusing on the use of mathematical language combined with everyday language and physical gestures to express mathematical content.

Through dialogue and direct observation, the research team found that the student had drawn illustrative diagrams and provided brief but unclear explanations. While speaking, the student pointed to the diagram and stated:

- "The second candy package is equal to the first candy package plus half of the third package".

- "The third candy package is equal to twice the first package plus the second package".

Although the student began to use sentences combined with simple illustrative diagrams and gestures, the explanations remained simple, incomplete, and unclear.

According to the criteria for requirement 3 in the rubric, the student achieved level 2.

For Exercise 4, the research team continued organizing group discussions and invited students to present their solutions on the board. The teacher observed and assessed the students' discussion process and their presentation, focusing on their confidence in answering questions and presenting mathematical content.

In this exercise, student Le Hai Chau answered:

"I calculated the volume of the box and then divided it by the volume of the wooden block. I took 8 × 4 × 4 and divided it by 4 × 1 × 1".

The student used hand gestures to point to the rectangular box and used short sentences to explain her thinking. While the voice was relatively clear, it was sometimes hesitant, and the student did not consistently demonstrate confidence in presenting the solution.

According to the criteria for requirement 4 in the rubric, the student achieved level 2**.**

Based on the evaluations for each exercise, the research team assessed the student's level of mathematical communication competence as follows: Requirement 1**:** Level 4; Requirement 2**:** Level 2; Requirement 3**:** Level 2; Requirement 4**:** Level 2.

We further illustrate our analysis through the continued example of student Nguyen Gia Han. Through the written and oral responses provided by student Nguyen Gia Han, it is evident that the student demonstrated proficiency at level 4 for requirement 1 in the rubric. Moreover, for Exercise 1, Nguyen Gia Han presented the solution using short but meaningful sentences, for example:

- Item a), the student wrote: "120 students are divided evenly among 4 buses, so each bus has 30 students". Additionally, the student was capable of producing brief paragraphs or structured sequences of calculations to articulate their problem-solving process. In several instances, the student also posed and answered their own questions to clarify reasoning and provide more in-depth explanations. Specific examples include:

- Item b): "Each vehicle consumes 15 liters of gasoline. Therefore, 4 vehicles consume: 15 × 4 = 60 liters. With gasoline priced at 25,000 VND per liter, the total fuel cost is: 60 × 25,000 = 1,500,000 VND".

- Item c): "First, I calculated the total amount contributed by the students: 120 × 50,000 = 6,000,000 VND. Then I subtracted the fuel cost of 1,500,000 VND from part b), and the remaining amount was 4,500,000 VND". The student further reflected by asking, "If the fuel price changes, will the result also change?" and explained: "Yes, because fuel cost is a component of the total expense".

- Item d): "If the fuel price increases by 10%, then the new price becomes 27,500 VND per liter. The new total cost is: 60 × 27,500 = 1,650,000 VND. By subtracting the original cost, we find a difference of 150,000 VND.

According to the rubric, the student achieved level 3 for requirement 2.

In this task, the research team facilitated a group discussion and invited the student to present her solution at the board. The evaluation focused on the student's use of mathematical language, everyday language, and physical gestures to communicate ideas effectively.

Through observation and dialogue, it was noted that the student used illustrative diagrams to support her explanation, which showed improvement in clarity and structure. For instance, while pointing at the visual representation, the student explained: "First, I know that the first candy package weighs $\frac{1}{4}$ kg, so I drew a small segment to represent $\frac{1}{4}$ kg. The second package is equal to the first package plus half of the third, so I drew a large segment showing half of the third package, then added one more segment of $\frac{1}{4}$ kg. The third package equals twice the first plus the second, so I redrew the second package and added two small segments for the $\frac{1}{4}$ Kg, with the first segment indicating half of the third package".

According to the criteria for requirement 3, the student was assessed at level 3.

The research team continued the group discussion format and invited Nguyen Gia Han to present her solution. The focus was on evaluating confidence in answering questions and clarity in presenting mathematical content. The student responded: "This problem asks for the maximum

number of wooden blocks that can fit into a box. To solve it, I need to calculate the volume of both the block and the box. I chose this approach because it fits the problem's requirements and helps me find the correct answer. First, I calculated the block's volume by multiplying length × width × height = 4 cm^3. Then, I calculated the volume of the box: 8 × 4 × 4 = 128 cm^3. Finally, I divided the box volume by the block volume to find how many blocks fit: 128 : 4 = 32 blocks".

During the task, the student raised an inferential question: "Would changing the order of calculating the volumes affect the result?". This question was discussed among group members as they explored alternative approaches. When presenting, the student demonstrated clear and coherent articulation, logical and persuasive reasoning, and used multiple forms of expression, including verbal explanation and gestures. Notably, the student showed a high level of confidence throughout the exchange.

According to the rubric, the student achieved level 4 for requirement 4.

Based on the assessment for each exercise, the research team assessed the student's level of mathematical communication competence as follows: Requirement 1: Level 4; Requirement 2: Level 3; Requirement 3: Level 3; Requirement 4: Level 4.

Overall, the rubric for assessing mathematical communication competence in Grade 5 students serves as a valuable assessment tool that can be utilized at any stage of the teaching and learning process in mathematics. It contributes to improving instructional quality while supporting the timely development of students' mathematical communication competence.

3. Conclusions

The rubric for assessing mathematical communication competence developed in this study, when applied appropriately and flexibly in teaching practice, contributes to a clearer identification of 5th graders' levels of competence in receiving, presenting, exchanging, and responding to mathematical content. Preliminary testing results indicate that the tools effectively assess students' mathematical communication competence, while providing a basis for teachers to adjust instructional strategies to better align with the capacities and needs of individual learners. This contributes to enhancing the quality of mathematics education in accordance with the competence-based orientation emphasized in the 2018 National Mathematics Education Curriculum. In future research, we plan to refine and improve the toolkit, expand its application to other primary school grades, and experiment with diverse assessment forms, such as group activities, project-based learning, and student presentations.

REFERENCES

[1] National Council of Teachers of Mathematics (NCTM), (2000). *Principles and standards for school mathematics*.

[2] Muhammad M, Purwanto J & Christiyansyah O, (2021). Mathematical communication ability students rely on learning motivation during the COVID-19 pandemic. *Journal of Physics: Conference Series*, 1778(1), 012014. https://doi.org/10.1088/1742-6596/1778/1/012014.

[3] Ministry of Education and Training, (2018). *National Mathematics Education Curriculum.*

[4] ISODA M, (2008). Japanese problem-solving approach for developing mathematical thinking and communication: Focus on argumentation with representation and reasoning. *Proceedings of APEC-Khon Kaen International Symposium 2008 Innovative*

Teaching Mathematics through Lesson Study III - Focusing on Mathematical Communication, Khon Kaen Session, Thailand, 30-43.

[5] Sunardi S & Prastiti TD, (2023). The analysis of the improvement of students' mathematical communication skills in solving problems of circumference and area of circles under the implementation of inquiry-based learning. *International Journal of Current Science Research and Review*, 6(3)**.** https://doi.org/10.47191/ijcsrr/v6-i3-35.

[6] Agustina L, Nurdin A, Ansari R & Sukriyah Y, (2023). Analysis of students' mathematical communication ability with realistic mathematics education. *KnE Social Sciences*. https://doi.org/10.18502/kss.v8i4.12908.

[7] Vu TB, (2016). *Fostering mathematical representation and mathematical communication skills for students in teaching mathematics in grades 6 and 7*. PhD thesis, Vietnam Institute of Educational Sciences.

[8] Turner R, Blum W & Niss M, (2015). Using competencies to explain mathematical item demand: A work in progress. *Assessing mathematical literacy,* 85-115. Springer, Cham. https://doi.org/10.1007/978-3-319-10121-7_5.