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INTEGRATING MATHEMATICS AND ECONOMICS WITH THE SUPPORT OF GEOGEBRA SOFTWARE IN TEACHING THE SYSTEM OF FIRST-ORDER INEQUALITIES WITH TWO UNKNOWNS AT THE HIGH SCHOOL LEVEL

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Abstract. The 2018 General Education Curriculum has introduced numerous innovative teaching methods including integrated teaching. Within the context of integrated teaching, interdisciplinary integration is among the teaching methods that have drawn significant attention. Meanwhile, the content of the system of first-order inequalities with two unknowns is a crucial topic in mathematics, potentially enhancing the integrated teaching methods. This study introduces a new teaching approach that integrates interdisciplinary teaching of mathematics and economics with the support of GeoGebra software in teaching the system of first order inequalities with two unknowns at the high school level. Building on existing research on integrated teaching, the research team employs both theoretical and practical research methods to develop a teaching approach tailored to the current educational context in Vietnam. The research has identified two approaches for exploiting, providing the teaching process as well as outlining methods to organize interdisciplinary teaching of mathematics and economics through economics problems about the system of first-order inequalities with two unknowns. The research focus of the article represents a significant contribution to enhancing the quality of teaching and learning.

Keywords: integrated teaching, interdisciplinary, system of first-order inequalities with two unknowns, mathematics, GeoGebra, economics.

1. Introduction

Over the past 5 years, since the introduction of the 2018 General Education curriculum, numerous new teaching methods have emerged, among which integrated teaching is the teaching method that garnered significant attention from many educators. Previously, the curriculum was organized in isolation. However, the approach of teaching subjects in isolation presents a significant challenge in education, preventing students from recognizing and connecting

knowledge and skills between subjects [1], [2]. Integration in teaching gives students the opportunity to experience and see the connection as well as the interrelationship between different contents of a particular curriculum [3].

Interdisciplinary integrative teaching is an approach that proposes situations that can only be logically approached through the illumination of multiple disciplines. In other words, subjects are integrated to solve a given situation. The learning processes will not be mentioned in isolation but must be linked together around the problems to be solved [4], making learning meaningful regardless of the student's grade level [5].

Numerous studies have been conducted on integrated teaching and interdisciplinary integration. However, interdisciplinary integrated teaching of mathematics and economics for the problem of the system of first-order inequalities with two unknowns in high school has not been mentioned in any works. Therefore, the introduction of a teaching method that integrates mathematics and economics for the problem of the system of first-order inequalities with two unknowns is a gap that needs to be filled in by researchers in our country. In order to fill the gap, the authors conducted research focusing on key issues, including the theoretical foundations of the integrated teaching perspective, interdisciplinary integration, integrated classification, comparisons between integrated teaching and traditional methods, teaching strategies that integrate mathematics and economics, the teaching process that incorporates mathematics and economics through specific problems, and the organizational structure for teaching according to this approach.

2. Content

2.1. Different views on integrated teaching

Integrated teaching encompasses a variety of views and perspectives. The term “integrated” is derived from the Latin word, which means the coordination of different activities to ensure harmonious functioning. Integration is best demonstrated by our own bodies, where many different systems exist and all work in synchronicity and rhythm. Integrated teaching is defined as the organization of teaching content of each separate subject connected into a unified whole [6].

In the educational aspect, integration is the process of making connections between skills and knowledge from resources and teaching experiences. It is the connection between theory and practice as well as the use of different perspectives to help students understand the problem comprehensively [7]. This is an activity that helps students connect old and new knowledge and understand the connections in the curriculum [8].

In 2019, in a study addressing the integration of mathematics, information science, and physics, we proposed a definition of integration. Integrating an object or phenomenon within a subject involves connecting that object or phenomenon to related phenomena, as well as identifying internal connections within the object itself. This integration forms a cohesive and meaningful unit within a specific context [9].

An essential skill of the 21st century is how to make sense of today's enormous and highly complex information. For this reason, it is important for students to develop the ability to connect scattered information. Thinking that combines scattered information can be defined as integrative thinking [10].

Integration is a teaching method that helps learners discover, collect, process, filter, and present information on related topics, the process of making connections between existing concepts and experiences in order to receive new information and successfully solve new problems [6].

From the aforementioned definitions, it is evident that integration represents a learning model that connects distributed information with both prior and newly acquired knowledge. However, it is not just about connecting scattered information but the unification between different parts to create a complete block to make learning complete, valuable, and stimulating student learning. In addition, integration depends on the cultural environment and the context of the teaching activity. People in one particular region will have a different way of receiving integrated teaching from those in other regions. People in one certain century will have a different approach to integrated teaching than people in other centuries. Referring to integration, we need to associate it with certain areas: Which historical period? Which learning society? No form of integration exists independently; it is invariably tied to institutions, social contexts, and cultural backgrounds.

2.2. Classification of integrated teaching

Integration can be classified in many different ways. In terms of subject content, there are four approaches. The first is esoteric integration. This integrated approach is taught in pursuit of the proposed integration goals, prioritizing the content of each subject. The second type is integration from a transdisciplinary point of view. The content develops competencies common to many subjects, called transdisciplinary competencies. The third is integration from an interdisciplinary point of view (integrate interdisciplinary teaching), also known as inter-subject integration. The content is the application of information from a variety of subjects to solve a problem. The fourth type of integration is viewed from a multidisciplinary perspective, where content is examined through the lenses of various disciplines. For example, the same content of a system of first-order inequalities with two unknowns can be studied in terms of mathematics, computer science, or economics [4].

In terms of integration, there are two approaches: horizontal integration and vertical integration. Horizontally integrated teaching is the organization of subject topics across a given period. In this teaching process, students integrate two or more subjects in the transmission of curriculum knowledge. In other words, horizontal integration is two or more subjects taught simultaneously. These subjects are taught in a unified way, not separately. Vertically integrated teaching is a way of organizing topics between different subjects over an indefinite period. In this teaching process, students integrate two or more subjects in different phases of the curriculum. In other words, vertical integration is the integration between subjects traditionally taught in different phases of the curriculum [6].

2.3. Compare integrated teaching with traditional teaching

On the basis of the work of Mathur et al [6], we have edited, supplemented, and made a comparison table between integrated teaching and traditional teaching as follows:

Table 1. Comparison between integrated teaching and traditional teaching

Integrated teaching	**Traditional teaching**
Taught by many teachers of different subjects at the same time or taught by a teacher with professional expertise in many different subjects.	Taught by a qualified teacher of a certain subject.
Do not repeat the teaching content too much	Teaching content can be repeated in a spiral pattern.

Time is better spent and more effective than traditional teaching	Spending more time than integrated teaching
The connection in integrated teaching is better than traditional teaching	The connection in the content is quite difficult
Stimulate thinking and improve the ability to grasp knowledge	Knowledge cramming
Understand and absorb knowledge better.	Less knowledge absorption
The learning content is consistent with each other	Learning content is separated by different high and low levels
An approach that improves and develops interest in the subject being studied	May discourage student learning
Clarify the concept more.	There is some confusion in the minds of students about the subject.
Emphasis on understanding concepts and current issues	Emphasis on testing and results
Less confusion when information is distributed on the same platform	Different information on the same topic on different platforms causes confusion in students' minds.
Develop students' creativity.	No or little development in students' creativity

2.4. Concept of interdisciplinary integrated teaching of mathematics and economics with the support of GeoGebra software

On the basis of the views on integration and interdisciplinary integration, we believe that the combination of mathematics and economics, supported by GeoGebra software, involves connecting knowledge, content, and methods from both fields to create a cohesive and comprehensive approach to the problem. This unity and comprehensiveness is not merely a mechanical addition of separate mathematical and economic components, but rather the formation of a new field that illuminates both mathematics and economics through GeoGebra software. The selected issue in interdisciplinary teaching, supported by GeoGebra software, must be relevant to real-life situations, aligned with the Vietnamese social context, and connected to global perspectives.

2.5. The advantages and limitations of interdisciplinary teaching

We compare the interdisciplinary teaching of mathematics and economics, supported by GeoGebra software, in the context of teaching the system of first order inequalities with two unknowns, examining both forward and reverse directions.

Table 2. Comparing teaching methods that integrate interdisciplinary teaching of mathematics and economics supported by GeoGebra software in the context of teaching the system of first order inequalities with two unknowns in the forward and reverse directions

Advantages and limitations	**Forward direction**	**Reverse direction**
Advantages	The problem statement is tight and realistic. There are no artificial problems or simulated realities, as all referenced materials have been evaluated and validated by experts in mathematics and economics.	Help exploit and expand the problem. The same math problem can give rise to multiple practical scenarios, allowing students to develop creative thinking.
Limitations	The problem cannot be expanded if this process is strictly followed.	The emergence of a pseudo-realistic problem or a simulation of reality, depends on the mathematical model in use. In other words, it is uncertain whether the economic problem we have presented can be translated into real-life situations.

2.6. Teaching process that integrates interdisciplinary teaching of mathematics and economics supported by GeoGebra software in the context of teaching the system of first order inequalities with two unknowns in high school

Building on the two teaching methods that integrate interdisciplinary teaching of mathematics and economics with the support of GeoGebra software in teaching the system of first-order inequalities with two unknowns at the high school level, we combine them into a unified teaching process consisting of four steps:

Step *1. Find the economic problem of the system of first order inequalities with two unknowns suitable for integrated teaching.*

(An economics problem typically refers to a situation or question that involves the allocation of resources, production, distribution, or consumption of goods and services).

Search in textbooks and references for problems that are often presented on systems of first-order inequalities with two unknowns framed as economics problems, such as finding the largest and smallest values of an objective function. These economics problems typically use common terms that are not overly specialized, making them both accessible for students and practical for teaching purposes. Teachers choose problems intentionally so that they can be developed and expanded in an integrated direction.

Step *2. Solve the economic problem.*

Model economics problems through mathematical frameworks. Use mathematical concepts found in textbooks to solve problems related to finding the largest and the smallest objective function. The objective function reaches its optimal value at the vertices of the polygon.

Step 3. Solve and verify economic problems with GeoGebra software

Use GeoGebra software to draw polygons. Create a Slider tool that moves the line representing the objective function to different positions to identify the maximum or minimum

value. Compare the results obtained through GeoGebra software with those derived through mathematical methods. Test both approaches and make corrections if any differences are found between the results from the two methods.

Step 4. Develop into another economic problem

The model of the system of first-order inequalities with two unknowns of the mathematical problem can be adapted to various economic problems. When utilizing and expanding this approach, it is essential to ensure the validity and practicality of the economic problems posed, avoiding scenarios where the data does not reflect real-world conditions.

2.7. How interdisciplinary teaching of mathematics and economics, supported by GeoGebra software, can be effectively organized for teaching the system of first-order inequalities with two unknowns at the high school level

Step 1. *Find the economic problem of the system of first order inequalities with two unknowns suitable for integrated teaching.*

We use the following problem in the Math Textbook 10, Volume One, The Horizon of Creation series:

Problem 1

Uncle Nam plans to grow corn and green beans on a piece of land with an area of 8 hectares. If he plants a hectare of corn, it takes 20 working days and yields 40 million dongs. If he grows a hectare of green beans, it takes 30 working days and earns him 50 million dongs. How many hectares does uncle Nam need to grow for each type of plant to get the most money? Given that Uncle Nam can only use no more than 180 working days to grow corn and green beans [11].

Step 2*. Solve the economic problem.*

- Teacher: Call x the number of hectares of land planted with corn and y the number of hectares of land planted with green beans. We have the following constraints with x, y:

+ Obviously $x \geq 0, y \geq 0$.

+ The cultivated area does not exceed 8 hectares so what will we have?

- Students: $x + y \leq 8$

- Teacher: The number of working days used does not exceed 180 so what will we have?

- Students: $20x + 30y \leq 180$

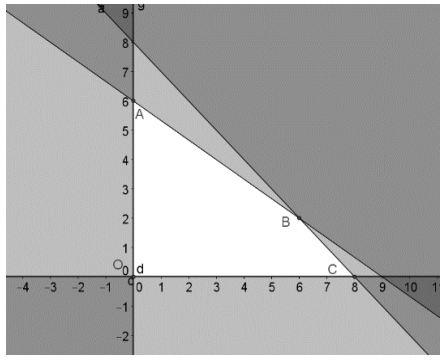
- Teacher: What will we have in the system of inequalities?

- Students: From that, we have a system of inequalities describing the constraints:

$$\begin{cases} x + y \leq 8 \\ 20x + 30y \leq 180 \\ x \geq 0 \\ y \geq 0. \end{cases}$$

- Teacher: Representing the solution domain of this system of inequalities on the coordinate axes Oxy

- Students: we get the quadrilateral domain. $OABC$.



- Teacher: Find the coordinates of the vertices of the quadrilateral.

- Students: The coordinates of the vertices of the quadrilateral are

$O(0; 0)$; $A(0; 6)$; $B(6; 2)$; $C(8; 0)$.

- Teacher: Call F The amount of money (in million dongs) uncle Nam earned, what will we have?

- Students: F = 40x + 50y

- Teacher: We must find *x, and y* that satisfy the system of inequalities in order for *F* to reach maximum value, meaning that we need to find the maximum value of the expression F = 40x + 50y on the quadrilateral domain *OABC.* Calculating the values of the expression F at the vertices of the polygon, what will we have?

- Students:

At O(0,0) : F = 40.0 + 50.0 = 0 at $A(0; 6): F = 40 \cdot 0 + 50 \cdot 6 = 300;$

At B(6;2): F = 40.6 + 50.2 = 340 at $C(8; 0): F = 40 \cdot 8 + 50 \cdot 0 = 320.$

- Teacher: What will the *F* reach maximum?

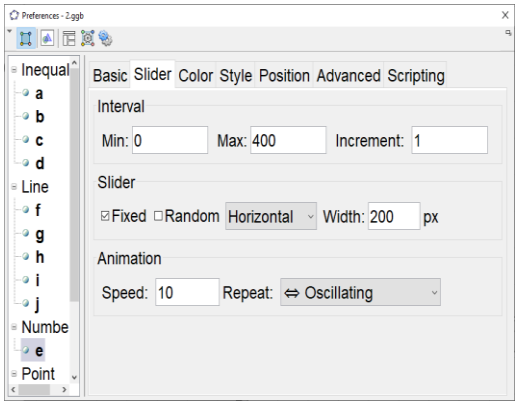
- Students: F reaches a maximum value equal to 340 at B(6;2).

- Teacher: Please give the conclusion.

- Students: To get the most money, uncle Nam needs to plant 6 hectares of corn and 2 hectares of green beans [11].

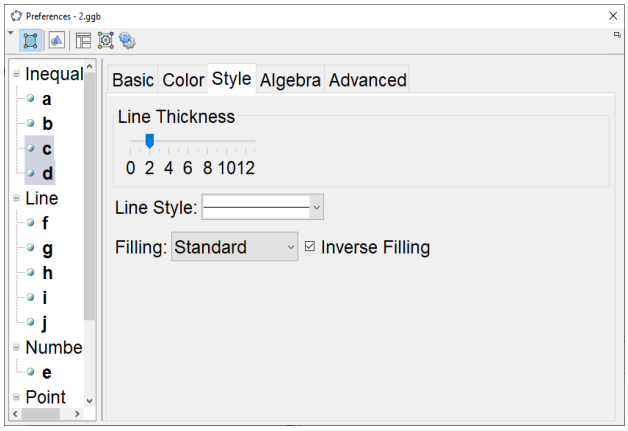
Step 3. Solve and verify economic problems with GeoGebra software

- Teacher: Click on the Slider command button Slider and enter the slider's parameters e as follow:



- Teacher: Enter the input box: x + y $\leq$ 8 and then to the domain x + y $\leq$ 8. Click on the box Inverse Filling (Because the solution domain representation on GeoGebra is opposite to the

solution domain representation in the textbook, you must click on this box to make the two solutions the same).



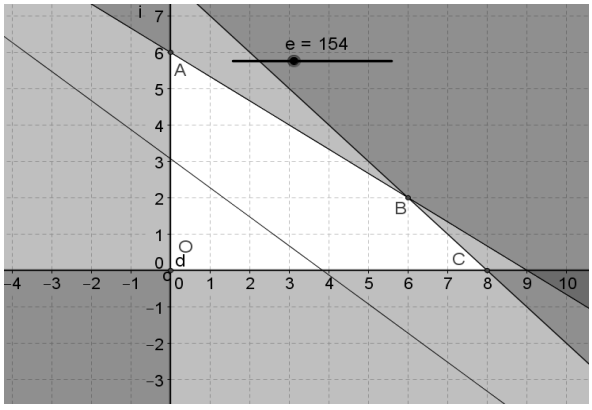
- Teacher: Similarly, input the inequalities: $20x + 30y \le 180$; $x \ge 0$; $y \ge 0$ and click on the box *Inverse Filling* of these inequalities respectively.

- Teacher: Enter the input box: $e = 40x + 50y$ we get a straight line with the equation:

$$f : 40x + 50y - e = 0.$$

- Teacher: Enter the input box in these straight lines:
$x + y = 8$; $20x + 30y = 180$; $x = 0$; $y = 0$.

- Teacher: Call the intersection of $20x + 30y = 180$ and $x = 0$ is A; $x + y = 8$ and $20x + 30y = 180$ is B; $x + y = 8$ and $y = 0$ is C; $x = 0$ and $y = 0$ is O. We get the following interface:



Move e on the slider then the line f will move parallel to the line $40x + 50y = 0$. Observing the values, we can see e reaches its maximum value when the line f still cuts the quadrilateral $OABC$ at 340. Now the line f go pass $B(6; 2)$.

To get the most money, uncle Nam needs to plant 6 hectares of corn and 2 hectares of green beans.

Students: Pay attention to the teacher.

Step 4. Develop another economic problem.

- Teacher: Derived from the mathematical model:

$$\begin{cases} x + y \le 8 \\ 20x + 30y \le 180 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

We propose some other economic problems as follows:

Problem 2:

A person uses two types of materials to produce two types of products P and Q. In order to produce 01 kilogram of each product P or Q, he must use a number of kilograms of different materials. The total number of kilograms of materials of each type that the person has and the number of kilograms of each type of material required to produce the product of each type are given in the following table:

Material Type	**Number of kilograms of materials available**	**The number of kilograms of each type of material required to produce 1 kg of product**	
		P	*Q*
A	8	1	1
B	80	20	30

Given that 1kg of product *P* yields 40 USD and 1 kg of product *Q* yields 50 USD. Make a plan to produce the two products above to get the maximum profit.

Problem 3:

An enterprise can use two types of raw materials $N_{1\ and}$, N_2 to produce a product according to two different technologies CN_1 and CN_2 The total volume of raw materials of each type that the enterprise currently has, the consumption rate of each material type per hour produced by each technology and the output (product/hour) for each technology are shown in the table below:

Material	**Total volume available**	**Consumption rate per hour**	
		CN_1	CN_2
N_1	8	1	1
N_2	180	20	30
Products		40	50

Make a plan so that the total number of products is maximum.

- Students: Pay attention to the teacher.

3. Conclusions

Our article has presented perspectives on integrated teaching. Integration is the connection between old knowledge and experience with new ones as well as the connection between content and subjects together to create a unity in a certain cultural and social context. In addition, the article also discusses methods of classifying integration, highlighting various types of integration classifying methods. Classifications based on subject content include inter-subject, interdisciplinary, trans-disciplinary, and multi-disciplinary teaching, as well as horizontal and vertical classifications. In the article, we give our own conception of the interdisciplinary integrated teaching of mathematics and economics for the problem of the system of first-order inequalities with two unknowns at the high school level. This concept is the premise for us to

introduce two teaching methods that integrate interdisciplinary teaching of mathematics and economics for the problem of the system of first order inequalities with two unknowns. One is the integrated teaching of mathematics, information, and economics in the forward direction and the other is interdisciplinary teaching in the opposite direction. On the basis of two forms of interdisciplinary teaching in mathematics, information, and economics, we propose an integrated teaching process for the interdisciplinary teaching of mathematics and economics, specifically focused on the system of first-order inequalities with two unknowns at the high school level. Additionally, we provide examples illustrating how to integrate mathematics, information, and economics in teaching in accordance with this teaching process. In the method of conducting integrated teaching, we use the Slider and buttons of GeoGebra software to help find solutions and verify problems. This is an advantage of integrated teaching that traditional teaching methods either lack or are rarely able to implement.

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