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THE ROLE OF METACOGNITION IN MATHEMATICAL MODELING PROCESS

Nguyen Thanh Cong[1,2]
[1]*PhD Candidate of Department of Mathematics, Ho Chi Minh University of Education, Ho Chi Minh city, Vietnam*
[2]*Bui Thi Xuan High School, Ho Chi Minh city, Vietnam*
*Corresponding author: Nguyen Thanh Cong, e-mail: ntcong179@gmail.com

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Abstract. The 2018 Mathematics General Education Curriculum includes the following cognitive activities as the components of the mathematical modeling process: recognizing mathematical models (such as formulas, equations, tables, graphs, etc.) for situations that arise in real-world problems; solving mathematical problems using the established model; presenting and assessing the solution in a real-world context; and improving the model if the solution is not appropriate. However, for students to carry out the aforementioned procedure, it's essential that they engage in a particular kind of activity called metacognition. Metacognitive activities in mathematical modeling encompass all of the learner's thinking that takes place throughout the mathematical modeling process. The article presents some research findings on the role of metacognition in the mathematical modeling process, including reading and understanding real-life problems, building mathematical models, solving math problems with the model, and presenting the solution.

Keywords: metacognition, mathematical modeling process, reading comprehension, strategy selection.

1. Introduction

The term "metacognition," also known as "awareness about awareness" or "thinking about thinking," was coined in 1976 by American psychologist John Flavell to characterize people's awareness of their own cognition [1]. Originally, metacognition was primarily explored in reading, but in the 1980s, metacognition started to be investigated in mathematics education, particularly concerning problem solving [2]. The first definition of metacognition can be traced to Flavell (1976): "Metacognition refers to one's knowledge concerning one's own cognitive processes and products or anything related to them (…). Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects on which they bear, usually in the service of some concrete goal or objective" [1], [3].

Several researchers have developed a number of models of metacognition in an effort to concretize the concept, namely J. Flavell's model (1979), Ann Brown's model (1987), Nelson and Narens's model (1990), Tobias and Everson's model (2002), etc. [4]. More recently, metacognition is typically classified into two categories: metacognitive knowledge, which is the awareness of one's past actions, and metacognitive strategies, which are the awareness of one's future actions. Metacognitive knowledge comprises declarative, procedural, and conditional knowledge as well as personal, task, and strategic knowledge. Metacognitive strategies include planning, monitoring - adjusting, and evaluating.

The mathematical modeling process, similar to the order of manifestations stated in the 2018 Mathematics General Education Curriculum, includes three steps: identifying the mathematical model (including formulas, equations, tables, graphs, etc.) for situations that arise in real-life problems; solving mathematical problems within the established model; presenting and assessing the solution in a real-world context and improving the model if the solution is not appropriate [5].

Numerous international studies have demonstrated the critical role that metacognition plays in both the capacity and process of mathematical modeling. Stillman (2011) highlights the significance of metacognitive activities, particularly in shifting from one stage to another in the modeling process [6], [7]. Blum (2011) states: “Metacognitive activities are not only helpful but even necessary for the development of modeling capacity” [8], [9]. Riyan Hidayat and colleagues (2018, 2023) proposed and tested the hypothesis that "Metacognition has a positive influence on students' mathematical modeling ability" [10], [11]. Katrin Vorhölater colleagues [4] introduced the idea of metacognitive modeling competencies, which encompass declarative meta-knowledge and procedural metacognitive strategies.

The current article presents some research findings on the role of metacognition in the mathematical modeling process as follows: reading and understanding real-life problems, building mathematical models, solving mathematical problems, and presenting solutions, through a task as follows:

"On a visit to Hanoi, Nam was attracted by the unique steel tubular arch construction of the bridge, which has three main spans with a width of 55m each, and 8 motor vehicle lanes (1). Please help Nam determine the maximum height of the arch that corresponds to the middle span of the bridge, given that the distance between the two feet of the middle arch is 77 m (2) and the height of the arch from the point on the bridge, away from the foot of this arch 16 m, is 12 m (3), given the tubular arches have a nearly parabolic shape”.

Figure 1. A bridge with parabolic-shaped arches

I chose this problem for two main reasons. First, there were several technical terms relating to construction and bridge building in the task, such as "bridge spans, steel tubular arch," and the students had to predict and guess the meaning of these terms when making models. Second, there were two assumptions: assumption (3) - “the height of the arch from the point on the bridge, away from the foot of this arch 16 m, is 12 m”, which was difficult for the students to understand, so they had more time to finish their own drawings to interpret the assumption; and assumption (1) - “a width of 55m, and 8 motor vehicle lanes (1)”, which was not related to the problem, which the students had to realize and simplify. The task provided multiple chances for students to conduct their metacognitive activities. The experiment was conducted on 112 grade 10 students at Bui Thi Xuan High School.

2. Content

2.1. Roles of metacognition in the mathematical modeling process

2.1.1. Roles of metacognition in reading and understanding real-life problems

When students begin to solve real-life problems, their first task is to read and comprehend real-life problems. Being first examined in reading [7], metacognition plays a role in helping students understand the problem. Specifically, metacognition facilitates learners to organize their reading, including what to read (the whole text, or some parts, or some keywords, etc.) and how to read (skimming, reading slowly, reading quickly, reading attentively, reading again, etc.). Occasionally, these activities happen naturally during reading, without the learner realizing that they are controlling it. Some researchers have pointed out the difference between good readers and poor readers: Baker (1989) found that “good readers appear to have better awareness and control of their own cognitive activities while reading than poor readers” [12], [13]; Long and Long (1987) reported that “good comprehenders engage in mental interactions with the text through visualization, self-questioning, and inferring, although poor comprehenders engage in some metacognitive activities, such as skimming, rereading, and pointing to keywords. They perform behaviors similar to those of good comprehenders, but without mentally activating operations needed for understanding” [12].

Furthermore, some researchers have proposed various reading comprehension strategies: organizational strategies, contextual thinking, reflective thinking, and imagery strategies by Jone, Amiran, and Katim (1985) [12]; skimming, predicting, visualizing, checking understanding, clarifying, self-checking content, reviewing, summarizing, activating prior knowledge, connecting previously knowledge to new information; etc. by Hartman (2001) [12].

In addition to reading text, metacognition also plays a role in helping learners interpret the meaning of images [14], because in some real-life problems, there may also be pictures, images, or illustrations. Sometimes, looking at images also makes it easier to absorb and comprehend text more quickly. In the proposed problem, there are two assumptions relating to construction and building including "steel pipe arch” and “bridge span"; therefore, it can be challenging for learners to interpret and determine their meanings without the picture.

2.1.2. Role of metacognition in building mathematical models

In this phase, learners need to identify variables, parameters, and constants (with conditions) and establish relationships between variables (cognitive activities). Metacognition is important in helping learners make appropriate decisions and alter decisions if the past decision is found inappropriate, for example: how many variables to set (one, two, or three variables), which object to set as the variable (object asked in the question or another object), which mathematical operations (+, – ×, ÷), power or exponentiation, logarithm, etc.) and which comparison operations

($>,<,\geq,\leq,=,\neq,...$) to use, which shape is similar to the object in the problem (triangle, rectangle, square, circle, rectangular box, etc.). The choice may be made based on students’ experience, preference, or even the benefits and drawbacks of that choice. For example, sometimes, a problem can be solved by setting one or two variables, and students can choose based on their preference. Determining which object to set as a variable is also important, because if students select an inappropriate object, the model may get too complex to be solved. In that case, students must change the way they set the variable to establish a more simple model.

2.1.3. Role of metacognition in solving math problems and presenting solutions

In this phase, metacognition helps learners break the problem down into steps (what to do first, what to do next) and choose the solution strategies by evaluating the pros and cons of each strategy. Metacognition assists learners to present, display, and justify the solution by organizing and arranging the layout of the solution, including determining which parts need to be written first and which parts need to be written later. Each student has a different layout for presenting the solution.

2.2. Experiment

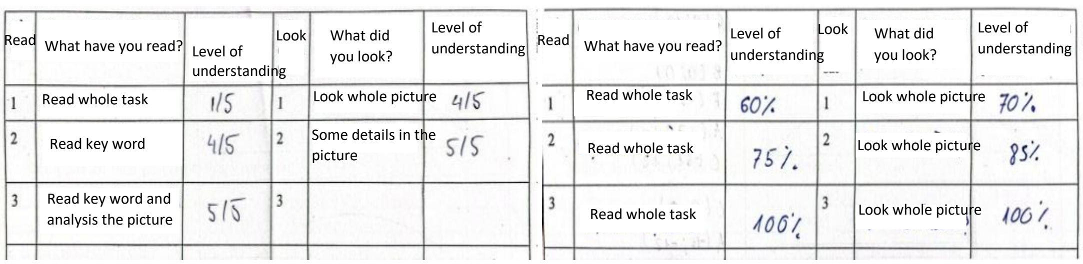
2.2.1. Experiment 1 (corresponding to the first role)

Hartman (2001) raised the question "To what extent do students understand what they read in a text?" [12]. To find out the answer, we designed a small experiment for students to self-assess their reading comprehension of the Bridge problem.

Table 1. Students' self-assessment of reading comprehension worksheet

Reading text	**What have you read?**	**Level of understanding**	**Looking at images**	**What did you look at?**	**Level of understanding**
First time	Read the whole task.		First time	Look at the whole picture.	
2th time			2th time		
3rd time			3rd time		
...	...	...	...	...	...

The experimental results showed that the students had different ways to express their level of understanding: using words (81 out of 112 students) such as "not understand yet, understood, somehow understand, understand a little, understood slightly, understood completely, understand almost completely, not clear yet, understand clearly, generalize, temporarily understand, etc."; using percentages (26 out of 112 students) for example 50%, 60%, 100%, etc., and using fractions (4 out of 112 students) such as 1/5, 4/5, 5/5, etc. Gradual increases in students' understanding after each time reading text or scanning images can be observed. Thereby, it is found that using percentages to evaluate the reader's level of understanding of the text is the most reasonable.



Read	What have you read?	Level of understanding	Look	What did you look?	Level of understanding
1	Read whole task	1/5	1	Look whole picture	4/5
2	Read key word	4/5	2	Some details in the picture	5/5
3	Read key word and analysis the picture	5/5	3		

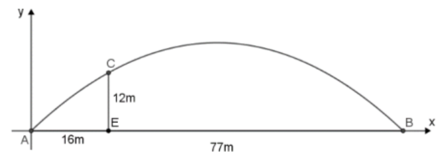
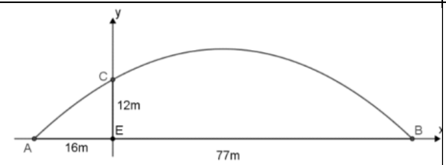
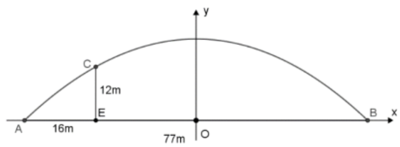
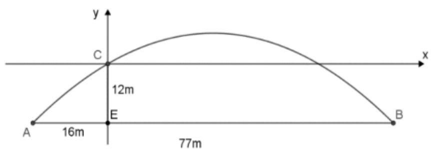
Read	What have you read?	Level of understanding	Look	What did you look?	Level of understanding
1	Read whole task	60%	1	Look whole picture	70%
2	Read whole task	75%	2	Look whole picture	85%
3	Read whole task	100%	3	Look whole picture	100%

Figure 2. Samples of students' completed worksheets

2.2.2. Experiment 2 (corresponding to the second role)

In the Bridge problem, the model chosen for the problem is a quadratic function $y = ax^2 + bx + c$, with the participation of two variables x, y, parameters a, b, c with the condition $a \neq 0$. So the coordination strategy has been implemented. The question is, where did the students put the coordinate system?

Table 2. Some strategies to locate the coordinate system and determine the coordinates of the three points

Strategy	Illustrative diagram	Determining the coordinates of points A, B, and C
Strategy 1		Simply, you can quickly determine the coordinates of three points A, B, and C;
Strategy 2		To determine point B, subtraction must be performed.
Strategy 3		Determining the coordinates of points A, B, and C requires complex calculations.
Strategy 4		Determining the coordinates of points A, B, and C is relatively complicated.

Each strategy has its own advantages and disadvantages. If the learner wants to determine the coordinates of points A, B, and C quickly and easily, the learner can choose strategy 1. If the learner prefers practicing more calculating skills by determining the coordinates of complex points, they can also opt for the remaining 3 strategies. In addition to the above 4 strategies, they also proposed a few other unusual alternatives.

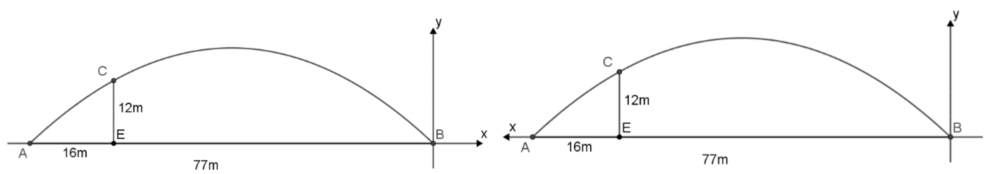


Figure 4. Students' unusual strategies

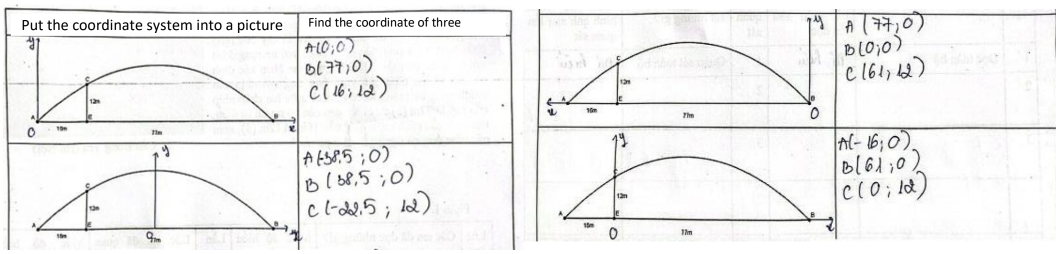


Figure 5. Sample of student's completed worksheet

Metacognition can be measured based on the number of strategies that the learners can propose: 13 out of 112 students couldn't propose any strategy to find coordinates, 1 out of 112 students can propose one way, 11 out of 112 students proposed two ways, 12 out of 112 suggested three ways, 76 out of 112 proposed four ways.

2.2.3. Experiment 3 (corresponding to the third role)

The procedure of solving the problem includes the following steps: (a) finding the coordinates of three points, (b) solving the system of first-order equations with three unknowns to find the a, b, c and (c) finding the y -coordinate of the vertex. There are several strategies to find y -the coordinate of the vertex: (1) using a calculator (the quadratic equation solving function), (2) applying the formula x = $\frac{-b}{2a}$ or (3) using a graph to find the x-coordinate of the vertex and then substitute it into the function, (4) directly applying the formula $y = \frac{-\Delta}{4a}$. Alternatively, students can (5) use technology support to find the highest point of the parabola by taking a photo of the cross section of the bridge, putting it into Math software, and using the figure to determine the result. Metacognition can be measured based on the number of strategies that the learner can propose. The experimental results showed that 23 out of 112 students could not propose a clear strategy, 17 out of 112 students listed one strategy, 45 out of 112 students listed two strategies, 23 out of 112 students listed three strategies, and 4 out of 112 students listed all four strategies.

Table 3. Worksheet: Listing some strategies to find the y-coordinate of the vertex

Choose a way to locate the coordinate system and write the quadratic function. Then, please list some strategies to find the coordinates of the vertex. Strategy 1: .... Strategy 2: .... Strategy 3: .... Strategy 4: ....

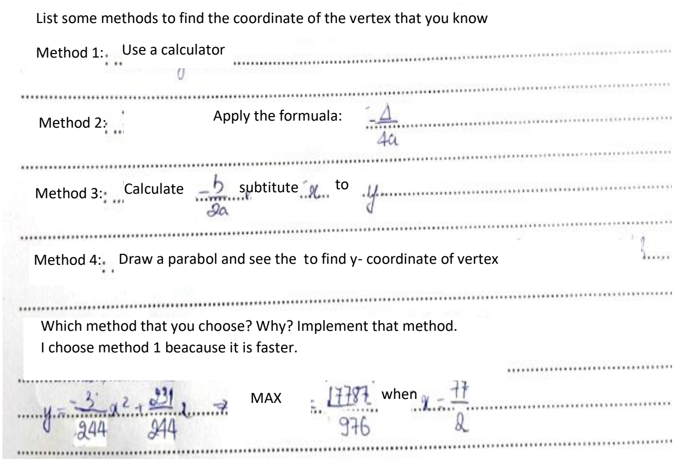

List some methods to find the coordinate of the vertex that you know
Method 1: Use a calculator
Method 2: Apply the formuala: $\frac{-\Delta}{4a}$
Method 3: Calculate $\frac{-b}{2a}$ subtitute x to y
Method 4: Draw a parabol and see the to find y- coordinate of vertex
Which method that you choose? Why? Implement that method.
I choose method 1 beacause it is faster.
$y = \frac{-3}{244}x^2 + \frac{231}{244}x \Rightarrow$ MAX $\frac{17787}{976}$ when $x = \frac{77}{2}$

Figure 3. Sample of student's completed worksheet

2.2.3. Experiment 4 (corresponding to the fourth role)

In the problem, the procedure to present the solution includes these steps: (1) drawing an illustration and attaching a coordinate system; (2) writing the general quadratic function; (3) determining the coordinates of the points; (4) solving a system of first-order equations with three unknowns; (5) finding the vertex coordinates; (6) concluding the height of the central bridge arch. The combination of different strategies to attach the coordinate system and many find the vertex coordinate of the parabola can produce different ways of presenting solutions to the problem (the learners only choose one of the options to present).

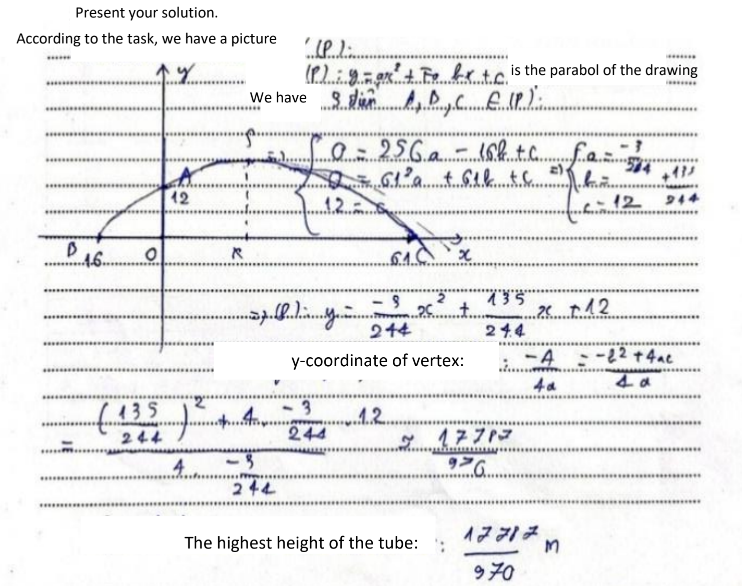


Figure 4. Sample of the student's complete solution

Some students (47 out of 112 students) did not know how to present the solution, 42 out of 112 students presented incomplete solutions (lacked drawings or conclusions, etc.), and 23 out of 112 students were able to present completely and correctly. Therefore, it can be seen that by presenting the solution, it is possible to evaluate the learner's mathematical modeling ability.

3. Conclusions

Metacognitive activities are crucial for the mathematical modeling process, helping learners with their reading to understand real-life problems, propose and select appropriate strategies during the problem-solving process (i.e. building a mathematical model), and eventually solve mathematical problems in the model, deciding the steps of solving strategies and the layout of the solution presentation, etc. Without metacognitive activities, the modeling process would be demanding for students, especially when they implement this process independently. Our research also opens up the approach to evaluate and measure learners' metacognition in the mathematical modeling process through the number of times they read the text and scan the image, the number of proposed strategies or solutions, and the ability to present complete solutions.

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