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THE GEOGEBRA SOFTWARE AS A TOOL TO FACILITATE THE CONSTRUCTION OF KNOWLEDGE ABOUT THE CENTER OF SYMMETRY OF A SHAPE AT THE 6TH-GRADE

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Abstract. In the 2018 curriculum for general education, the Geometry and Measurement module at the middle school level supplements visual geometry content. Teaching visual geometry, especially the concept of shapes with a center of symmetry, the center of symmetry of a figure at the 6th-grade level poses many challenges for teachers. This article presents a teaching design for the concept of shapes with a center of symmetry, the center of symmetry of a figure with the support of GeoGebra software. The "dynamic" features of the GeoGebra software have facilitated students in the process of acquiring knowledge and simultaneously created opportunities for students to form and develop mathematical competencies.

Keywords: shapes with the center of symmetry, the center of symmetry of the figure, GeoGebra, visual geometry, mathematical competencies.

1. Introduction

The Vietnamese Mathematics Curriculum 2018 (issued on December 26, 2018) has defined the content of Geometry and Measurement in secondary education, including Visual Geometry and Plane Geometry. According to the Ministry of Education and Training (2018), Visual Geometry in secondary education is defined as "continuing to provide language, symbols, descriptions (at a visual level) of objects in reality (plane shapes, solid shapes); establishing some common geometric models, calculating some geometric factors; developing spatial imagination; solving some simple real-life problems related to Geometry and Measurement" [1]. Although this shift in focus creates a stepping stone for students to understand Geometry and measurement more conveniently, it also poses a series of challenges for teachers in teaching. In the Visual Geometry curriculum in grade 6, students face challenges when learning the content of shapes with centers of symmetry, such as locating the center of symmetry of a shape, especially when the shape is complex or lacks clear visual representation, or difficulties in using geometric tools such as compasses or protractors to draw and verify the symmetry of the shape. Teachers encounter difficulties in finding effective teaching methods**,** such as combining theoretical instructions on the board with the use of static images from textbooks. Finzer & Nick (1998) argued that dynamic geometry software and dynamic manipulations have the potential to create a new approach to teaching and learning mathematics in school [2]. According to Nam, Thai, and Dien (2018), by using mathematical software such as GeoGebra and Geometer's Sketchpad, one can create visual representations, especially dynamic ones while developing a learning environment that integrates mathematics with real-life situations [3].

This study aims to answer the research question: How does the use of GeoGebra help students to understand the concept of the center of symmetry of a shape?

The study designs teaching situations for the concept of the center of symmetry of a shape with the support of GeoGebra software. The design models aim to address the challenges associated with teaching and learning this knowledge.

2. Content

2.1. Visual geometry

The primary objective of teaching is to accurately identify and understand students' cognitive processes. According to The Ministry of Education and Training (2018), "The process of children's geometric cognition must progress from the specific to the abstract, from intuitive visual images to abstracted, formalized geometric knowledge" [1]. During this process, from grade 1 to grade 6, students become familiar with geometry through visual images or concrete objects without an element of inference". In the new middle school mathematics curriculum, Geometry and Measurement are not regarded as tightly formalized Euclidean geometry based on axioms. Instead, it is an Euclidean geometry that is structured based on "intuitive" and empirical principles. Teaching visual geometry serves as a preparation and a transitional stage for teaching Euclidean geometry with axioms, creating a harmony between intuition and deduction. Thus, teaching Visual geometry has established a cognitive foundation, enhancing observation and description skills while facilitating the acquisition of comprehensive geometric knowledge.

2.2. Geometric cognition

The theory of the three modes of human knowledge representation - enactive, iconic, and symbolic- defines the key forms of representation in mathematics. According to J. Bruner (1966), learning is a cognitive process through three modes of perception (Learning modes) that he believes follow a sequence: Enactive (action-based); Iconic (visual imagery); and Symbolic (abstract representation) (Figure 1) [4].

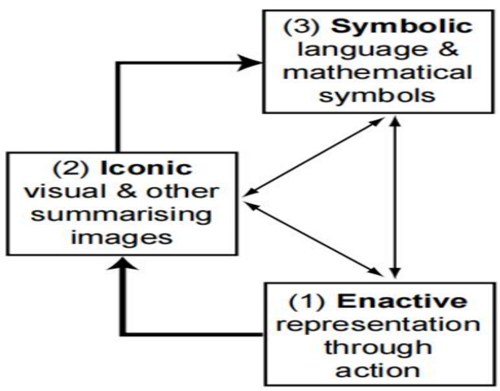

Figure 1. Bruner's three modes of representation

One effective learning process is the Concrete - Pictorial - Abstract (C-P-A) process.

The first stage in the C-P-A process involves teaching through concrete experiences, using real objects to introduce new concepts to students. By seeing and directly interacting with objects, students can better understand their properties and characteristics. For instance, studies by Carbonneau et al. (2013) found that students who use manipulatives could better understand math concepts than those who do not [5].

Pictorial: The second stage uses images or graphics to represent and visualize concepts. Students engage with images or diagrams to explore the relationships and correlations between concepts, which enhances their ability to visualize and understand what they are learning. Boaler (2016) suggests that visual representations help students make connections between physical experiences and symbolic representations, solidifying their comprehension [6].

Abstract: Finally, in the abstract stage, students are introduced to abstract expressions and symbols to represent the concept. After completing the concrete and pictorial stages, students will find it easier to grasp and understand abstract concepts.

The CPA approach is particularly effective in developing problem-solving competencies. A study by Witzel, Mercer, & Miller (2003) demonstrated that students with learning difficulties benefited from the CPA approach, showing significant improvement in their problem-solving skills [7]. By gradually abstracting problems, students develop a flexible mindset that allows them to approach mathematical problems from multiple angles. The National Council of Teachers of Mathematics (NCTM, 2000) emphasizes that this method helps in understanding the "why" behind mathematical procedures, which is crucial for tackling non-routine problems [8]. A study by Lambert & Sugita (2016) found that students who struggle with traditional teaching methods often excel when using CPA because it allows them to engage with math at their own pace and through various modalities [9]. Hinton et al. (2017) found that the CPA approach supports better long-term retention of mathematical concepts compared to direct instruction or rote memorization techniques [10]. Students who engage in the CPA sequence tend to remember mathematical concepts better and can apply them in new contexts, as the transition from concrete to abstract ensures a deeper understanding.

According to the Van Hiele theory (1986), there are five levels of thinking or understanding in geometry: Visualization; Analysis; Abstraction; Deduction; and Rigor [11]. Van Hiele identifies visualization as the first level of geometric cognition, which parallels the Concrete stage of the CPA approach. In both frameworks, initial exposure through tangible experiences is seen as essential for establishing a deeper understanding, thereby paving the way for more abstract reasoning in subsequent stages.

All three theories emphasize the development of cognition through stages from concrete to abstract. Bruner's theory and the C-P-A process share the idea that students need to progress through concrete stages before grasping abstract concepts. Van Hiele's theory also describes a similar process in the context of geometric cognition, from basic visualization to abstract logical reasoning [11]. Bruner and C-P-A focus on all areas of cognition, while Van Hiele specifies geometric learning but still follows the same model of cognitive development from concrete to abstract [4].

2.3. Challenges in teaching the center of symmetry of a shape

According to the Ministry of Education and Training (2018), the learning objective for teaching the central symmetry of a shape is to “Identify the center of symmetry of a plane figure. Recognize plane figures in the natural world that possess central symmetry (when observed in a two-dimensional image)” [1].

To identify the challenges teachers and students face in teaching and learning visual geometry, we surveyed 20 grade-6 mathematics teachers and 80 sixth graders in Ho Chi Minh City. For the survey, we used a questionnaire that included various types of questions: yes/no, a rating scale, and open-ended questions. The purpose of using these types of questions was to validate our preliminary assumptions while also exploring the thoughts of teachers and students about the challenges they encounter in teaching and learning central symmetry.

**** Survey results for teachers***

Question 1: Did you encounter difficulties in finding or using dynamic models to illustrate the concept of central symmetry? (Answer: Yes/No). The results showed that 100% of teachers had difficulties in finding suitable dynamic models. In further interviews, teachers mentioned that finding images was not difficult as there are many online sources or real-life examples. However, finding appropriate dynamic models was much more challenging. Some teachers expressed interest in using software like GeoGebra to create dynamic models but faced difficulties in generating model ideas and using GeoGebra's features.

Question 2: How would you rate the effectiveness of designing teaching activities in helping students recognize shapes with central symmetry? (Rating scale from 1-5). 95% of teachers rated the effectiveness at level 5, while 5% rated it at level 4.

Question 3: What is the biggest challenge in teaching central symmetry? (Open-ended question). The results indicated that identifying suitable ways to communicate with students was the main challenge. Some teachers further shared that the nature of visual geometry relies more on description rather than theorem-based presentation, requiring careful selection of language to ensure students' understanding.

Key challenges for teachers from the survey results are as follows:

(i) *Finding dynamic models to illustrate the concept of central symmetry*: Central symmetry is an abstract concept, and using dynamic models can help students better visualize it. However, designing these models requires creative ideas and flexibility in using software features, which can be demanding for teachers.

(ii) *Designing teaching scenarios that encourage students to build knowledge:* Designing exercises or real-life activities that allow students to explore and recognize shapes with central symmetry requires teachers to be both creative and knowledgeable about active learning methods. This can be challenging for teachers who are not yet familiar with such requirements.

(iii) *Identifying appropriate communication methods:* Each student possesses a unique approach to understanding, so it is also formidable to find teaching methods that are easy to grasp and suitable for the majority of students.

**** Survey results for students***

Question 1: Do you find it difficult to identify shapes with central symmetry? (Answer: Yes/No). The results showed that 70% of students found it difficult to identify shapes with central symmetry. Further interviews revealed that students could more easily recognize shapes with few patterns, but they struggled when the shapes were more intricate.

Question 2: Do the practical activities or exercises provided by your teacher help you understand central symmetry better? (Rating scale from 1-5). The results showed that 70% of students felt they understood the activities. However, when further interviewed, students mentioned that while arriving at an answer was not too difficult, explaining their reasoning was the more significant challenge.

Question 3: How would you like teachers to help you understand the concept of central symmetry better? (Open-ended question). The survey results indicated that most students felt that understanding the center of symmetry required more visual aids.

Key Challenges for Students Identified from the Survey:

(i) *Recognizing knowledge about shapes with central symmetry through observation and perception:* For many students, identifying shapes with central symmetry through observation alone can be challenging, especially when the shapes are complex, contain many patterns, or are not symmetrical. Improving this requires students to develop their geometric thinking and analytical skills.

(ii) *Building knowledge of central symmetry through activities:* Students need to engage in practical activities to gain a better understanding of central symmetry. However, without proper guidance, exploring the concept on their own can become ambiguous and confusing.

2.4. The role of GeoGebra

GeoGebra is a comprehensive dynamic mathematics software platform designed for all educational levels, integrating geometry, algebra, spreadsheets, graphing, statistics, and calculus into a single engine. It provides teachers with a valuable opportunity to create interactive online learning environments, offering students multiple avenues to explore the mathematical concepts being taught. GeoGebra software could be truly useful in dealing with the following activities: Demonstration and visualization media; Construction aids; and Discovery aids. As a visual tool, GeoGebra can demonstrate geometric concepts, and allow students to see real-time changes and relationships between shapes and figures, making abstract concepts more tangible through dynamic visualization. GeoGebra can function as a tool for constructing geometric shapes, lines, and other figures. It enables students to manipulate elements with ease, thereby facilitating a hands-on understanding of the construction processes and properties of geometric entities. The software is also beneficial for discovery-based learning activities, where students can experiment with different geometric properties and relationships, thereby encouraging exploration and hypothesis testing, leading to a deeper understanding of mathematical principles through guided discovery.

For students, GeoGebra has impacts on their learning: Visualizing: Students can see the abstract concept; Representations: Students can make connections; Experiments: Students can discover mathematics. To leverage the benefits of GeoGebra in teaching, the aforementioned aspects need to be thoroughly explored to create effective teaching practices.

2.5. Teaching design

In the Visual Geometry curriculum for sixth grade, the requirements regarding shapes with centers of symmetry and the center of symmetry of a shape are outlined as follows: Recognizing the center of symmetry of a plane shape; Recognizing plane shapes in the natural world that have centers of symmetry (when observed in two-dimensional images). To design teaching situations, we adhere to the following process: Design ideas and design steps.

Design ideas: We utilize interactive features to create dynamic representations. *Design steps*: We proceed in two steps:

Step 1: Draw symmetric shapes (line segments, circles, parallelograms).

Step 2: Create a motion path for the model.

From the design steps above, we created a model on GeoGebra and set tasks for students to interact with the model. To assist students in navigating the model with ease, we incorporated sliders which, once adjusted, students can observe visual representations of the model, thereby enhancing their understanding. This design sequence aligns with the cognitive processes outlined in Bruner's theory, the C-P-A process, and Van Hiele's theory.

Task 1: Figure with central symmetry

To construct a model that aids students to recognize point O as the center of symmetry of the circle, we conducted the following steps: Consider any diameter AB, when the student presses the play button (Figure 2), segment OA is folded back and coincides with segment OB. The image shows that point A on the circle, when folded, coincides with point B and also lies on the circle. Students can choose a different diameter. From this image, the teacher introduces O as the center of symmetry of the circle.

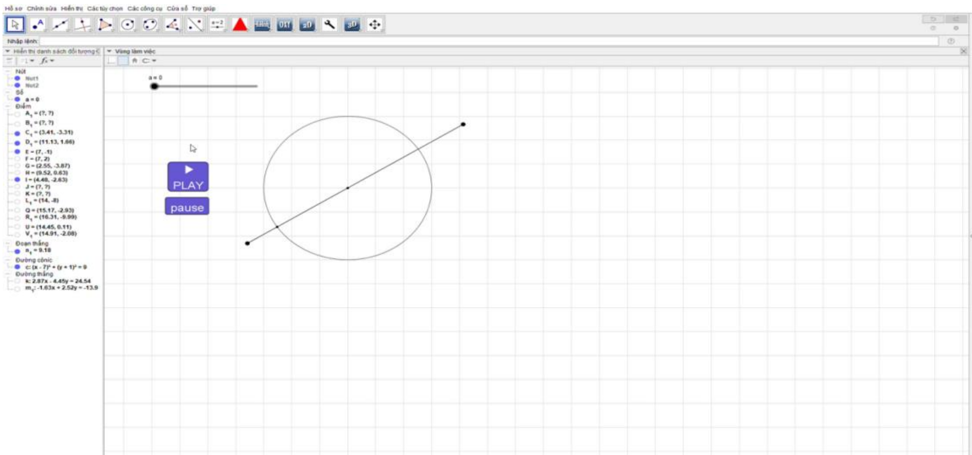

Figure 2. Circle model

Task 2: The central symmetry of a segment

For this task, we developed the model as follows: Initially, we created a line segment. As the slider moved, the left segment of the line would fold and coincide with the right half of the line (Figure 3). This visualization helps students realize that points symmetrical through the midpoint of the original line segment all lie on the original line segment.

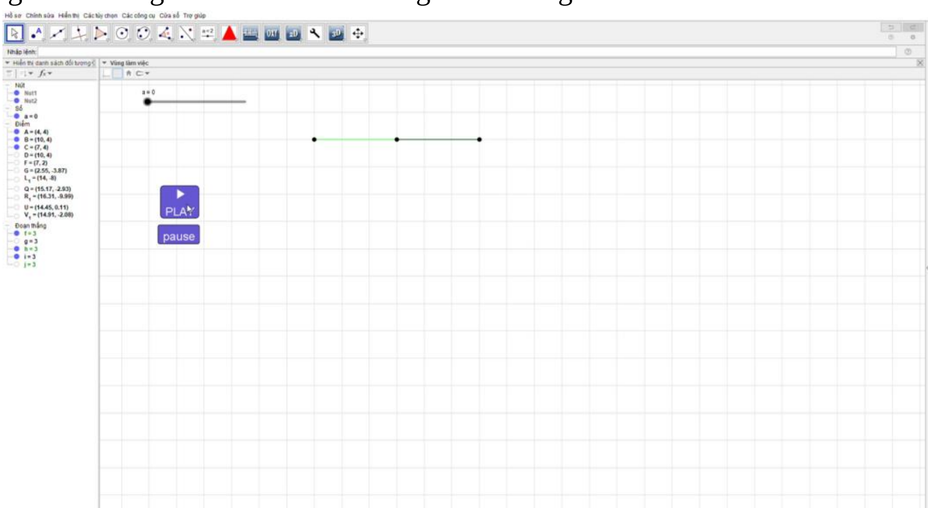

Figure 3. Segment model

Task 3: The central symmetry of a parallelogram

The following steps were implemented to construct a model to help students recognize the intersection point I of the diagonals of a parallelogram as its center of symmetry: Consider a straight line segment AB connecting two points on opposite sides and passing through I. When the student presses the play button, segment IA is folded back and coincides with segment IB.

The image shows that point A on the parallelogram, when folded, coincides with point B and also lies on the parallelogram (Figure 4). Students can choose a different line segment. From this image, the teacher introduces I as the center of symmetry of the parallelogram.

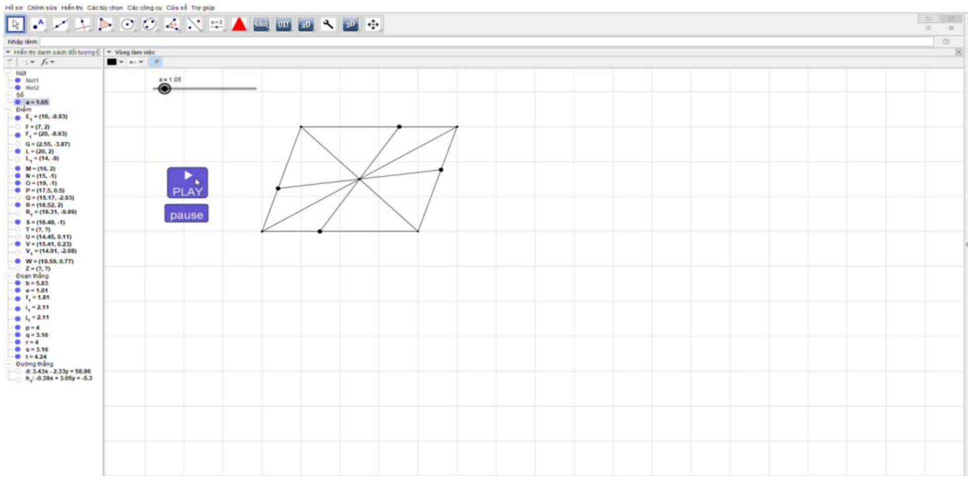

Figure 4. Parallelogram model

An experiment was conducted with 40 sixth-grade students at Lac Hong School, District 10, Ho Chi Minh City. The teacher followed a teaching process based on the designed tasks. In each task, the teacher introduced a dynamic model created using GeoGebra and then allowed the students to interact with the model. A few students manipulated the model directly while the entire class observed and answered questions for each task, thereby enhancing their understanding.

Regarding the instructional design, teachers commented that "the model has addressed the difficulties of accessing knowledge for teachers. Although the textbooks present the content quite clearly, explaining verbally or using 2D images from the textbooks is not easy." Additionally, "the model has greatly supported teachers in teaching. Teachers no longer need to use verbal explanations, and students' difficulties are addressed by letting them manipulate the model".

When studying with the model, most students’ responses were "I enjoy studying with the model because I can interact with software and dynamic images that help me understand the material more easily," and "I like it when teachers use the model during class because I get to learn about new software and I become more actively engaged in learning".

3. Conclusions

The study identified challenges in teaching and learning visual geometry, geometric cognition, and the role of GeoGebra in instruction. Building upon these foundations, we formulated design ideas and delineated the steps to design instruction on the center of symmetry of a shape and symmetric shapes. Effectively utilizing GeoGebra's features to create visual models for teaching the central symmetry of a shape necessitates that educators possess a comprehensive understanding of both GeoGebra and the cognitive processes involved in students' learning. Using the dynamic features in GeoGebra makes teaching central symmetry in geometry more effective. Dynamic representations not only deepen students' understanding of the concept but also actively engage them in the learning process. The interactive environment with models in GeoGebra promotes convenient knowledge construction and helps students develop self-learning skills effectively, positively contributing to the process of building knowledge for students.

REFERENCES

[1] Ministry of Education and Training, (2018). Mathematics Education Curriculum (issued together with Circular No. 32/2018/TT-BGDDT dated December 26, 2018 of the Minister). Retrieved from https://data.moet.gov.vn/index.php/s/m6ztfi7sUIIGQdY#pdfviewer.

[2] Finzer W & Jackiw N, (1998). *Dynamic Manipulation of Mathematics Objects*, Boston, USA: Key Curriculum Press.

[3] Nam PS, Thai LH & Dien DT, (2018). Orientations for Building Learning Situations in Mathematics to Link the Competency of Mathematical Problem Solving to Real Life, *Vietnam Journal of Education*, 5, 100-104.

[4] Bruner JS, (1966). *Toward a Theory of Instruction*. Cambridge: Harvard University Press.

[5] Carbonneau KJ, Marley SC & Selig JP, (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology*, 105(2), 380-400. https://doi.org/10.1037/a0031084.

[6] Boaler J, (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovative teaching*. Jossey-Bass.

[7] Witzel BS, Mercer CD & Miller MD, (2003). Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research & Practice*, 18(2), 121-131. https://doi.org/10.1111/1540-5826.00068.

[8] National Council of Teachers of Mathematics (NCTM), (2000). *Principles and standards for school mathematics*, NCTM.

[9] Lambert R & Sugita T, (2016). Increasing English language learner engagement in elementary mathematics: A focus on the CPA approach. *Bilingual Research Journal*, 39(3-4), 320-334. https://doi.org/10.1080/15235882.2016.1238904.

[10] Hinton V, Walker D & Tomlinson C, (2017). The CPA approach and its effect on retention in mathematics. *Journal of Mathematics Education*, 45(4), 215-230.

[11] Van Hiele PM, (1986). Structure and Insight: A Theory of Mathematics Education. Academic Press Inc.