HNUE JOURNAL OF SCIENCE
Educational Sciences 2024, Volume 69, Issue 5B, pp. 109-121
This paper is available online at http://hnuejs.edu.vn/es
DOI: 10.18173/2354-1075.2024-0139

ORGANIZING ACTIVITIES FOR PRIMARY STUDENTS TO EFFECTIVELY LEARN MATHEMATICAL VOCABULARY

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Received June 20, 2024. Revised October 20, 2024. Accepted December 28, 2024.

Abstract. Using mathematical language is of great significance to students in the process of learning mathematics. Improving students' mathematical language skills contributes to improving their mathematical learning outcomes. This process must be done continuously throughout the learning process. Mathematical vocabulary is an important part of mathematical language, the mastery of which fosters students to enhance their ability to use mathematical language accurately and effectively, thereby forming their mathematical logic senses. This article presents, based on basic mathematical language activities, a number of teaching organizing measures in which teachers support students to comprehend and use vocabulary effectively in learning and in practice.

Keywords: mathematical vocabulary, mathematical language, teaching mathematics in primary school.

1. Introduction

The foundation of mathematical language is mathematical vocabulary. Vocabulary is an integral factor in learning and mastering mathematical concepts. Developing sufficient vocabulary is an important goal in teaching mathematics, especially for students with poor language skills, who likely have inefficient problem-solving skills, an important skill in learning mathematics. In school, students who lack a foundation of vocabulary knowledge would struggle in the task of reading, writing, verbalizing mathematical concepts, and solving problems necessary for learning mathematics. Understanding and using vocabulary correctly is crucial to students' effective Mathematics learning. Therefore, organizing language activities to help students learn effectively is essential in teaching mathematics in general and in primary schools in particular.

Tran Ngoc Bich (2013) studied students' vocabulary knowledge in the first grade in the direction of developing effective reading skills concerning (1) Organizing student activities to form mathematical language knowledge including syntax and semantics of mathematical language; (2) Training students to use mathematical language in teaching concepts, rules, properties and solving problems [1]". Thai Huy Vinh (2016) focused on the development of mathematical language skills for primary school students in grades 4 and 5, specifically, in order to:

(1) Introduce and develop vocabulary knowledge for students; (2) Make students understand semantics associated with the correct use of mathematical language syntax; (3) Train students' skill to convert natural language to mathematical language and vice versa" [2].

It can be found that the mathematical language in primary school, especially mathematical vocabulary, has been explored by multiple researchers. Yet, there is still a gap for a specific way to help primary school students comprehend, memorize, and apply mathematical vocabulary effectively in learning mathematics that calls for adequate research actions.

2. Content

2.1. Some features of mathematical vocabulary

Hoang Chung (1994) believes that: "Mathematical language only includes mathematical symbols and terms" [3]. Not stopping there, mathematics educators in Vietnam continue to research mathematical language and add "diagrams, drawings, charts, graphs, etc." as components of mathematical language. The concept of mathematical language is expanded by researchers such as Le Van Hong (2003), Tran Ngoc Bich (2013), Vu Thi Binh (2016): "Mathematical language is the language of science, including a set of mathematical symbols, symbols (diagrams, drawings, charts, graphs, etc.), mathematical terms (words, phrases) and a set of rules for combining them to express, represent, notify, demonstrate and communicate objects and mathematical relationships while speaking, writing or thinking." [1], [4], [5]. Pham Van Hoan (1981) states that in teaching mathematics: "Spoken language is used in combination with visual language in textbook films, in audio tapes, etc." [6], thus, images are the objects of mathematical language, too.

When considering the relationship between mathematical vocabulary and mathematical concepts, the two authors Eula Ewing Monroe & Robert Panchyshyn (1995) divide mathematical vocabulary into 4 types: (1) Mathematical terms; (2) General vocabulary; (3) Symbols - signs; (4) words with multiple meanings even in mathematics" [7].

Thus, mathematical vocabulary is a set of words, terms, signs, symbols (diagrams, drawings, charts, etc.), and mathematical images used to describe, represent, and express mathematical content.

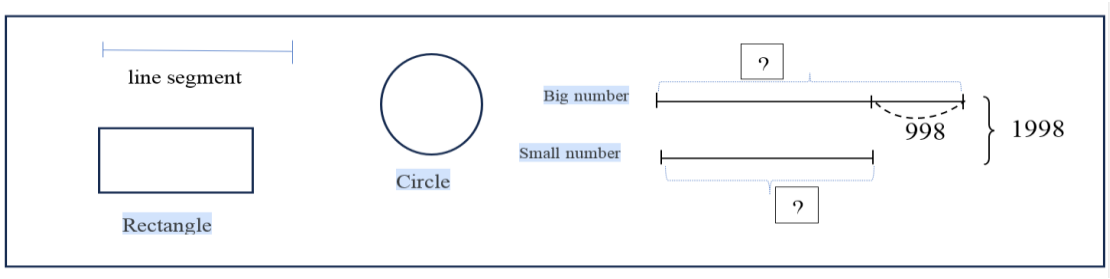
Mathematical terms refer to words and phrases that represent concepts, mathematical objects, and relationships in the field of mathematics, for example: even numbers, odd numbers, straight lines, right angles, denominators, numerators, etc.

Mathematical symbols include digits and symbols (+, :, …). Digits are the most common mathematical characters, representing numbers, and they are highly abstract. In addition, a specific digit can be used to express many different meanings. For example, consider the digit 2 in the following numbers:

$$13.25 \quad 325 \quad 82 \quad \frac{1}{2} \quad \frac{2}{3} \quad 15\text{m}^2$$

The digit 2 has different meanings (digit, power, denominator, numerator, unit of area) and is highly abstract in each mathematical content it represents. In addition to the ability to provoke complexity, the same meaning can be presented with different characters.

Mathematical symbols include diagrams, drawings, charts, graphs, etc. used to represent, express, and demonstrate mathematical relationships and specific mathematical objects. Mathematical diagrams are drawings that describe certain characteristics of objects, relationships, and mathematical content. In elementary school mathematics content, there are line diagrams, diagrams, models, drawings, charts, etc.



Mathematical imagery is a language expressed through images containing mathematical content, relationships, and objects without using words or text characters. For example, below is a mathematical image appearing in the Grade 1 Math textbook, "Canh Dieu" series [8].

Figure 1. Images in Grade 1 Math Textbook, "Canh Dieu" book series

The vocabulary system of a language is the basic component of that language. A vocabulary system along with its meanings is indispensable for the proficient use of a certain language. Mathematical vocabulary is essential in communicating mathematical ideas and concepts, the mastery of which is vital to children's mathematics learning.

2.2. Mathematical language activities in teaching mathematics in schools

Nguyen Ba Kim (2007) affirms that: "Each teaching content is related to certain activities; Language activities are one of the activities that appear in teaching Mathematics content in schools. Language activities are conducted when asking students to state and explain a certain definition or proposition, especially in their own words, or transform them from one form to another" [9]. Accordingly, the author points out that concept teaching activities, it is necessary to "pay attention to guiding and encouraging students to express definitions differently, in their own words".

When researching mathematical representations, Vu Thi Binh (2016) proposes 3 forms of mathematical language activities: "(1) Receiving mathematical language activities in terms of vocabulary, syntax, and semantics, etc; (2) Activities to convert ideas into words; (3) Language conversion activities from different forms of cognitive language, translating from natural language to cognitive language and vice versa" [5]. Le Van Hong (1998), when considering mathematical language in an expanded sense, along with the view that Mathematical discourse (spoken or written text) is a multi-signal genre, presents 4 types of mathematical language activities: "(1) Converting Mathematical content and ideas into a "physical shell", using signals from mathematical language; (2) Identifying mathematical content from certain mathematical languages; (3) Adjusting linguistic forms to keep the content and mathematical ideas intact; (4) Transforming signals of mathematical language, especially the signal of mathematical symbols, while still retaining parts of the content of mathematical ideas".

Based on the processes of teaching typical mathematics situations at the primary school level, including the formation of mathematical concepts, mathematical rules, formulas, algorithms, and mathematical problem solving, etc., we propose the following mathematical language activities:

(i) Acquiring mathematical language in terms of vocabulary, syntax, and mathematical semantics accurately, logically, and systematically.

Mathematical vocabulary at the elementary level is the first mathematical language equipped for students in the precise sense of mathematics. In addition, vocabulary is highly specialized, including terms (words and phrases), symbols (drawings, diagrams, charts, etc.), and visual language. The syntax of mathematical language is the rules for combining symbols and mathematical terms into mathematical formulas, expressions, and propositions. The semantics of mathematical language is understood as the meaning or content of symbols, terms, and mathematical symbols. For example, the term "Four plus two equals six" when taught to first graders:

Example 1. When teaching the lesson "Addition within 6" (Math 1, "Canh Dieu" Textbook Series) [8].

- Syntax: 4 + 2 = 6.

- Semantics: 4 plus 2 equals 6 or 4 plus 2 equals 6.

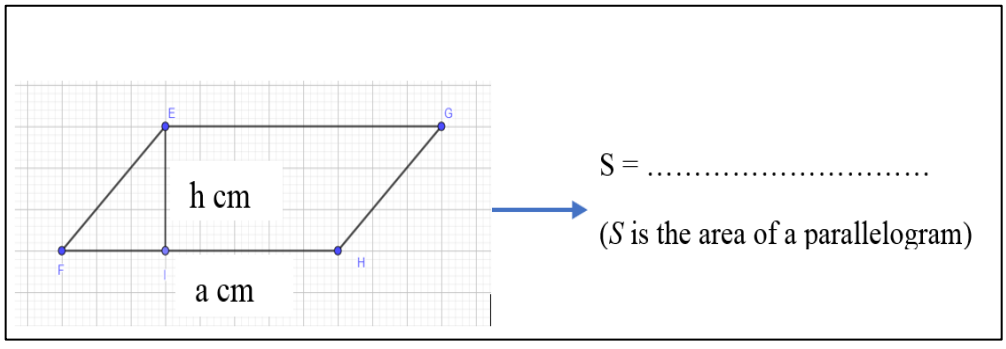
Attention to the process of forming and training students to understand and use mathematical words, terms, and symbols in definitions, rules, properties, and formulas, and to express mathematical propositions correctly and logically contributes to more effective mathematical operations. When students receive mathematical language using this activity, it helps them learn about the ideas of mathematical content that the mathematical language expresses through vocabulary, semantics, and syntax accurately, logically, and systematically.

(ii) Activities of expressing and describing mathematical content based on a specific mathematical language.

Each unit of knowledge in the textbook is initially presented through images, diagrams, drawings, then mathematical symbols, and mathematical terms, so the activity of describing and expressing mathematical content with images, diagrams, and drawings is critical to students' process of learning mathematics. When stating mathematical rules and properties, the very first step to support student's understanding and application is to describe and express those rules and properties using symbols or specific syntax at specific values.

For example: When 4th graders learn the rule for calculating the area of a parallelogram, teachers can organize the following pedagogical activity to instruct students to describe the mathematical content from a certain mathematical language.

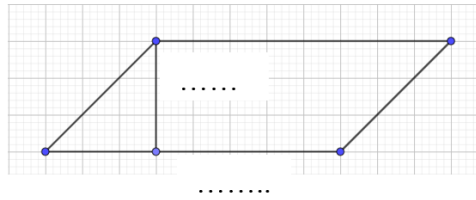
Example 2. Fill in the blank with the appropriate symbols.



or

$S = a \times h$

(S is the area of a parallelogram)



The task of expressing, explaining, and describing mathematical concepts and symbols requires students to mobilize knowledge and use language in general and mathematical language in particular in accordance with the context, ensuring the correct connection between types of mathematical language. In this activity, students recall the symbols and terms indicating the concept that needs to be expressed and described, determine the characteristics of the concept, and then state and describe it correctly. Furthermore, students can link concepts together, consider their similarities, and state the definition of the concept, describing this object through a similar object (maybe partially similar) to them.

(iii) Language conversion from different forms of mathematical language, conversion from mathematical language to natural language, and vice versa.

In each concept, rule, and mathematical property, natural language is always used in combination with terms, and symbols. Each statement (spoken) has many corresponding expressions in writing and vice versa. In this activity, mathematical language is the main means combined with natural language to create knowledge exchanges in mathematics learning activities. The activity of expressing and stating mathematical rules and properties in many different ways not only helps students remember and understand the nature of mathematical concepts, rules, and properties but also helps students practice using natural language more effectively while improving their ability to learn mathematics.

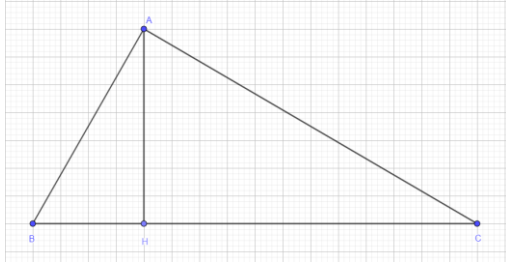
Take the following activities as some examples.

Example 3. When teaching 5th-grade students, teachers can organize students to state the rules for calculating the area of a triangle in the following ways:

Option 1: The area of a triangle is calculated with the base multiplied by the height (same unit of measurement) and then divided by 2.

Option 2: If S is the area of the triangle; h is the height corresponding to the base a, then $S = \frac{a\ h}{2}$.

Option 3. If



then

The area of triangle ABC is

Example 4. When teaching the concept of the rectangle to 3rd graders, we can organize students to make statements as follows:

+ A rectangle is a quadrilateral with 4 right angles and has two equal long sides and two equal short sides.

+ A rectangle is a parallelogram with one right angle.

When organizing students to perform conversion or "translation" activities using mathematical language, between mathematical language and natural language, it is necessary to ensure "precision", and strict adherence to mathematical language, and at the same time highlight the role and relationship of mathematical language forms as well as the flexibility of these language forms.

Teaching students how to "translate" from mathematical language to natural language and vice versa helps students form and practice specific mathematical knowledge and skills, but also helps students have the ability to apply mathematics effectively in practice, and at the same time see the meaning of mathematics in life.

(iv) *Transformation with the signal of mathematical language, mainly signals of mathematical symbols, while still keeping intact part of the content of mathematical ideas*.

According to the research results of Ton Nu My Nhat (2013): "the largest use in mathematical discourse is not language but symbolic signals. Instead of language, symbols are the means to present relationships quickly, concisely, and most economically" [10]. Thus, we cannot do math if we only use language; in other words, we cannot do math if we do not use drawings and mathematical symbols. Therefore, performing transformations with the signal of mathematical language is the process of training and forming students' ways to use mathematical language in the process of learning mathematics.

For example, when teaching students about the system of measurement units of a certain quantity, to establish a new relationship between measurement units, teachers can conduct the following unit conversion exercise:

Example 5: Fill in the blanks:

12345cm = ……. dm = …….. m =…….km

100m = ….. cm = …….km = …..

Creating and transforming exercises is an activity in teaching and solving math problems in primary school. Creating and transforming exercises helps students develop their generalization ability and problem solving skills, and practice their creativity in learning. In this activity, students use language to express tasks, construct new tasks, continue writing to comprehend new tasks, etc. from the original problem by changing data (usually a vocabulary with the original properties, or a solid word with a new property, or a new vocabulary with a completely new property, etc.).

Example 6: Given the task: "From the digits 1 and 2, write 4 two-digit numbers".

From the above problem, the teacher can organize students to perform the following activities:

Activity 1: Setting up a similar problem.

Activity 2: If the requirement of the problem is: "Write a natural number with 3 digits.", based on the given problem, write a problem for this question.

Activity 3: In the given problem, replace the condition "From the digits 1 and 2" with: "the sum of the two digits of the number to be found is 3". So how can the problem be stated?

Set up a new problem from a given problem by changing some conditions to have a similar problem, exploiting and changing conditions to have a new problem related to setting up numbers

with more challenging requirements to help students flexibly and effectively use mathematical language in learning.

Obviously, mathematical language activities in the above teaching situations in primary school involve mathematical language both as a tool, a means, and a result of mathematical language activities in which mathematical language fully demonstrates its functions in independent learning and communication activities.

2.3. Some main measures to help students learn mathematics vocabulary effectively in teaching mathematics in primary schools

2.3.1. Designing activities for students to identify and describe mathematical vocabulary through the process of teaching mathematical concepts

A mathematical concept is made up of connections between words, phrases, terms, and conjunctions in mathematics. Therefore, the process of forming mathematical concepts is also the process of identifying and defining mathematical vocabulary. Based on the language activities carried out in teaching mathematics in elementary schools, it's recommended to help students identify and describe mathematical vocabulary through the process of forming mathematical concepts by asking students to perform the following activities:

- Reading, writing correctly, and describing the characteristics of the term.

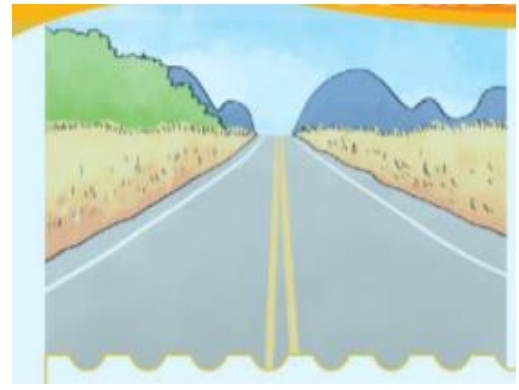
- Restating the concept of the term in as many ways as possible.

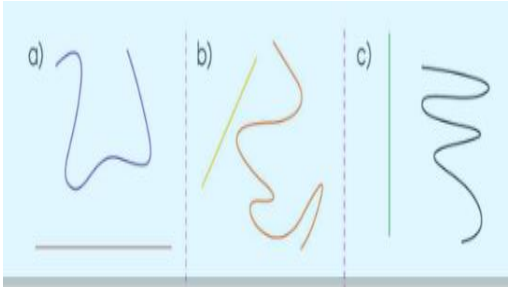
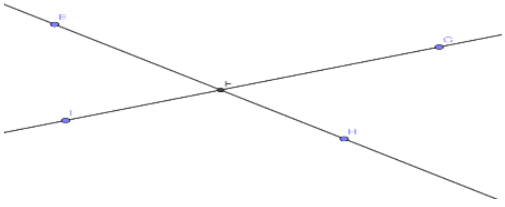
- Filling in sentence fragments and blanks with appropriate words, terms, and phrases to complete a concept definition.

- Explaining to confirm (or refute) the statement of properties related to a concept or a given object.

For example, in the lesson "Straight lines, curves, broken lines" (Math Textbook Grade 2, volume 1, page 86, Canh Dieu series), we can organize some following activities when introducing the terms "Straight line", "Three collinear points".

Example 7

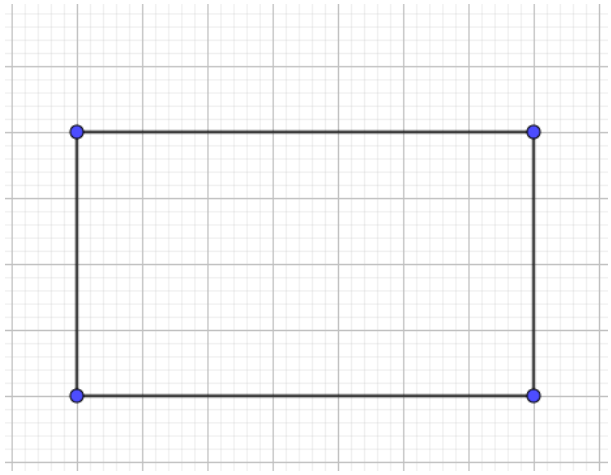
Activity 1: Name and describe the picture below.	 (I*mages in Math Textbook 1, "Canh Dieu" book series)*
Activity 2: The teacher draws a straight line on the board, students observe and the teacher asks students to name the object.	

Activity 3: The teacher formalizes the term "Straight line" and asks students to memorize, read, and write the term.	Straight line
Activity 4: The teacher asks students to identify the straight line through exercise 1.	Exercise 1: Point out straight lines.  (I*mages in Math Textbook 1, "Canh Dieu" book series)*
Activity 5: The teacher shows the next image and asks students to discuss what they recognize when observing the above image. (The student may describe: three points A, B, and C are on the same straight line, point B is between points A and C, etc.)	
Activity 6: The teacher formalizes the term: Three points in a straight line are collinear.	Three points A, B, C lie on the same straight line. Three points A, B, C are collinear.
Activity 7: The teacher organizes students to identify a straight line, three collinear points, etc. through exercise 2.	Exercise 2: Name three points on a straight line 
Activity 8: The teacher asks students to draw a straight line, showing three collinear points, and restate the characteristics of three collinear points.	

The language activities taking place in Example 7 include: Naming the terms associated with the symbol, describing the definition of the new object, drawing a line, and representing 3 collinear and non-collinear points.

Example 8. When students learn about Rectangles, we can organize the following student activities:

Activity 1: Name the shape in the picture.



Activity 2: List everything you know about this picture.

Activity 3: Name some real-life objects in the classroom that have this shape.

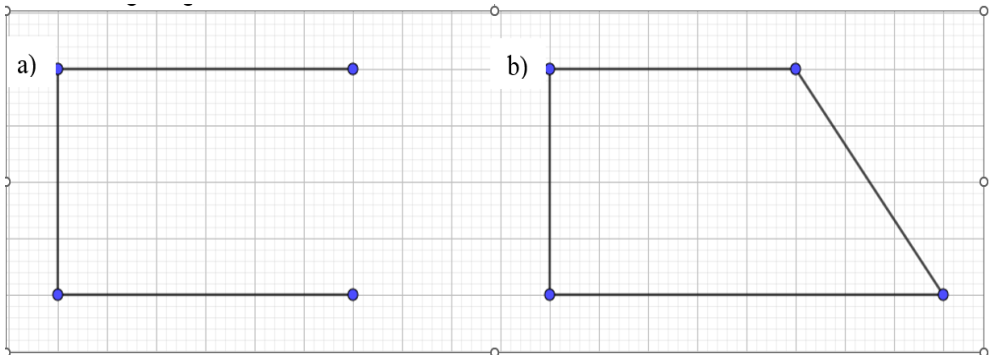
Activity 4: Explain why you chose the objects in Activity 3.

Example 9: After introducing the concept of a rectangle, to consolidate the knowledge of shape symbols, the teacher can organize the following activities:

Activity 1: In the following figures, which is a rectangle and which is not a rectangle? Why? (Given a set of many geometric shapes).

Activity 2: How many rectangles are there in the picture? Can you name those rectangles? (There are many shapes in one picture).

Activity 3: Draw another straight line to make a rectangle.



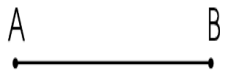
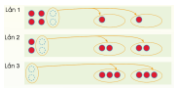
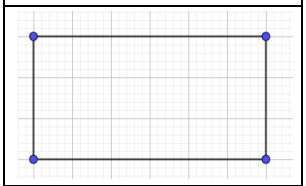
In this example, students can identify the symbol of the concept, the name of the concept (term), and explain the requirements of the problem.

These problems create opportunities for students to perform language activities compatible with the process of forming mathematical concepts in which students perceive, recognize, and use mathematical vocabulary.

2.3.2. Designing activities for students to identify and represent mathematical content in many different forms

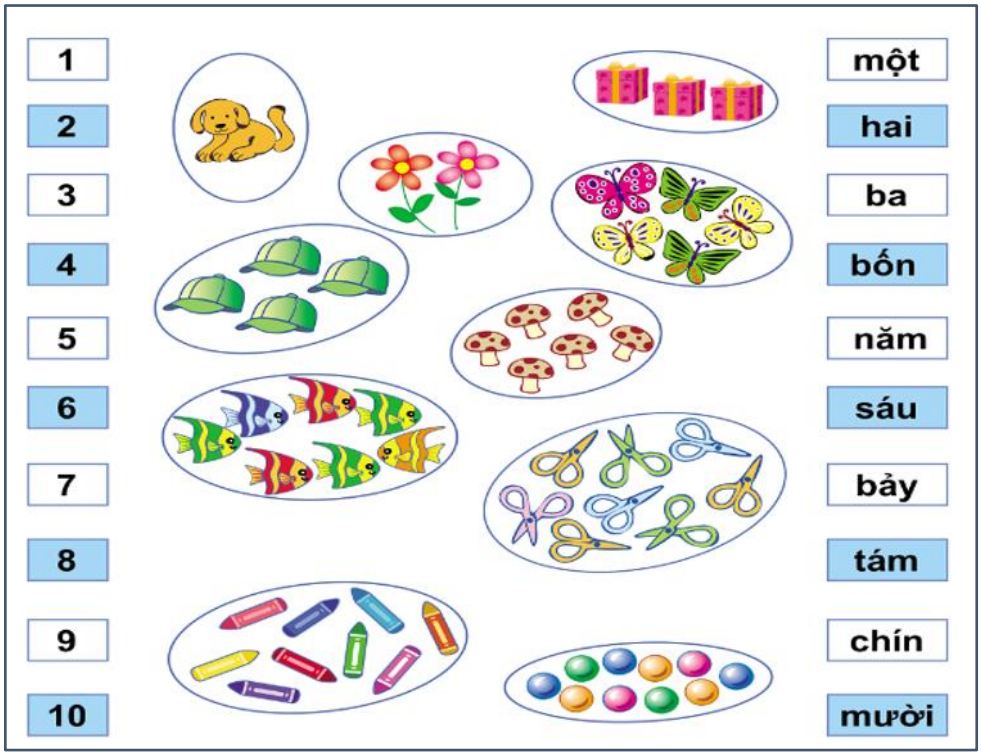
To represent a mathematical object or a mathematical relationship, we can use mathematical language in many forms: symbols, images, terms, and mathematical symbols.

Example 10. Match the appropriate symbols with the correct terms or mathematical images.

Symbol		Term		Mathematical Images		Mathematical symbols
3 + 2 = 5		Two				
100		One Hundred				
2		3 plus 2 equals 5				
6 : 2 = 3		6 divided by 2 equals 3				
		Rectangle				

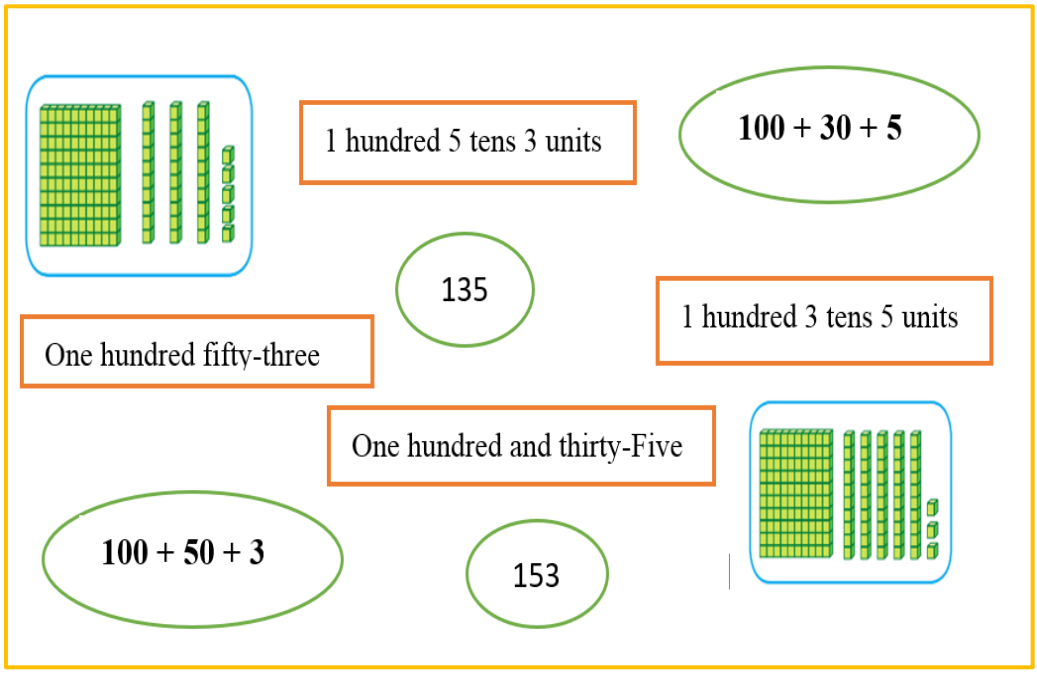
(*Images in Grade 1, 2 Math textbook, "Canh Dieu" book series*)

Example 11. Find the cards that have the same quantity.



(*Images in Math textbook Grade 1 - Connecting knowledge book series*)

Example 12. Match the cards with the same meanings.



(*Images in Grade 2 Math textbook, "Canh Dieu" book series*)

2.3.3. Designing activities for students to use vocabulary through solving mathematical problems

Mathematical problem solving is a fundamental activity in teaching Mathematics in primary schools. When solving math problems, students use the vocabulary system along with grammar rules to express mathematical content accurately through activities such as reading and understanding the content of the problem, developing solutions to the problem, and presenting the solution. Through solving math problems, teachers help students practice using mathematical language and expand the meaning of mathematical vocabulary. With the following activities, it is possible to create opportunities for students to use mathematical language, and consolidate and expand mathematical vocabulary in teaching Mathematics in primary schools.

- Summarize the problem in many different ways.
- Fill in the blanks with the appropriate words, terms, phrases, etc. to get the correct solution.
- Identify incorrect solutions and correct them.
- Solve the problem in many different ways.
- Detect problems with the same mathematical structure.
- Identify, state, and create new math problems.

Problem: Binh has 10 notebook labels. Binh has 3 more notebook labels than An. How many notebook labels does An have?

Hoa's solution	*Nga's solution*
Solution	Solution
The number of An's notebook labels is:	An's total notebook labels are:
10 – 3 = 7 (notebook labels)	10 + 3 = 13 (notebook labels)
Answer: 7 notebook labels	Answer: 13 notebook labels

Please choose the correct solution. Why did you choose that solution?

Example 13: Who do you think is correct?

Hoa: 20 – 8 : 4 × 2 = 6

Binh: 20 – 8 : 4 × 2 = 16

Hang: 20 – 8 : 4 × 2 = 19

Example 14. Read the problem and write the appropriate calculation in the blank spaces.

Classes 3A and 3B participate in a tug of war game, class 3A has 25 students and class 3B has 23 students. The number of students participating is divided equally into 4 teams. How many students are there in each team?

Solution

The number of students in classes 3A and 3B is:

? ? ? = ? (students)

Each team has the number of students:

? ? ? = ? (students)

Answer: ? students

Example 15. Choose the sign (+, -, ·, :) or parentheses () in the appropriate positions so that the calculations have the correct value:

a)

8 ? 4 ? 2 = 4

8 ? 4 ? 2 = 1

b)

8 ? 4 ? 2 = 10

8 ? 4 ? 2 = 30

3. Conclusions

In this article, we aim to propose some useful measures that can be implemented in the process of teaching Mathematics to primary school students to help students acquire some

mathematical vocabulary effectively. Understanding vocabulary provides students with a mathematical foundation, helping them improve their ability to learn Mathematics in general and primary school Mathematics in particular. Therefore, mathematical vocabulary needs to be taught actively and proactively throughout the learning process. We hope that some of the measures presented above if implemented well by teachers, will contribute to improving students' learning of Mathematics.

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