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THREE PHASES OF TECHNOLOGY IN SINGAPORE MATHEMATICS EDUCATION

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Abstract. This paper discusses the changes in classroom instructions due to technology over the years in mathematics education, and how these changes have impacted mathematics learning and teaching. The impact on learning can be seen over a few phases in Singapore: The use of scientific and graphing calculators has allowed the focus on the developing of higher-order thinking skills, while at the same time de-emphasizing routine computation. With the introduction of various computer software such as spreadsheets, mathematics teaching, and learning have moved towards the next level of emphasis on coding and computational thinking. Technology can and has been harnessed by teachers to enhance student learning. These will be discussed in detail in the talk, with particular reference to the Singapore education context.

Keywords: Technology in education; scientific calculator; problem solving; online platforms; flipped learning; generative AI.

1. Introduction

The emergence of technology into the education landscape has resulted in the evolution of the mathematics curriculum development and its enactment in the mathematics classroom. Such changes due to technology could manifest in many forms, for example, an additional tool to enhance learning through engaging students in more concrete visualization of concepts, a shift of focus of learning from low-order procedural knowledge to higher level conceptual understanding, or more radically, alter the content of coverage in addition to pedagogical changes. In this paper, we discuss the changes through the development of mathematics classrooms in the Singapore education system.

2. Content

2.1. Phases of technology

2.1.1. Phase one

Prior to the adoption of calculators in mathematics classrooms, slide rules have been used to perform various mathematical operations such as the four basic operations, finding roots and logarithms efficiently, among many other operations (e.g., [1], [2]). Much of the use of slide rules pertains to helping students with tedious computations, although there are other creative possibilities with such an instrument. For example, [2] demonstrated creative use of various mathematical formulae

using the slide rules. Still, this was not the norm but an exception of the day. The days of slide rules were soon followed by the introduction of handheld calculators and scientific calculators.

Handheld calculators were first invented by two companies, namely Canon in Japan and Texas in the United States, in 1970 [3]. The two companies have "successfully developed the world's first pocketable battery-driven electronic print-out calculator with full large-scale integrated circuitry." Scientific calculators were soon invented by Hewlett Packard in 1972. Scientific calculators were able to calculate the values of transcendental functions such as trigonometry or logarithmic functions. The latter scientific calculator soon found its place in the school system and subsequently had wide adoption in many parts of the world, including Singapore.

With the widespread adoption of scientific calculators in the early 1980s in Singapore secondary schools, slide rules went behind the scenes. Almost at the same time, scientific calculators were allowed in the high-stake national examinations at the secondary level in GCE 'O' Level mathematics. The objective of using calculators was to enable the students to focus on the mathematical concepts rather than the mundane calculations (https://stationeryworld.com.sg/blog/tips-guidelines-on-using-calculators-in-the-national-exam#:~:text=Scientific%20calculators%20were%20first%20 introduced,device%20as%20an%20examination%20aid).

There have been concerns about students' use of calculators. In the study carried out by [4], the main concerns among teachers on early use of calculators by children include the worry that children could become over-reliant on the tool without understanding the mathematical processes; children might not learn the fundamental facts; or there might be unequal opportunity of access to calculators among students. There have been many studies on the impact of the use of calculators on students' learning of mathematics. According to meta-analyses carried out by [5]-[7], students using scientific calculators had a positive attitude towards the learning of mathematics. However, [6] shows that early use of calculators (e.g., in Grade 4) hindered the development of basic computational skills among average students. The prevalence of this stream of thought continued to sustain resistance to the use of calculators for younger children. Scientific calculators were only allowed for secondary school students up to 2008 when younger children in primary 5 and 6 (ages 11 and 12) were allowed to use the scientific calculators in classrooms and during their assessment.

With the emergence of the Singapore mathematics curriculum framework in the early 1990s, the role of the calculator could then be articulated to support the curriculum. Figure 1 shows the centrality of mathematical problem solving in the Singapore mathematics curriculum from K-12. This framework explicates the objective of mathematics education, which is flanked by five attributes, represented by the five sides of the pentagon, contributing to mathematical problem solving [8].



Figure 1. The first Singapore mathematics curriculum framework that appears in 1990 [8]

Readers should note that the framework in Figure 1 remains largely unchanged, despite the regular curriculum revision that has been carried out since then. One can easily make a comparison between the framework when it was first introduced into the curriculum (Figure 1) and the most recent mathematics curriculum framework as it stands today (Figure 2). The fundamental structure of the framework, including the centrality of problem solving and its five attributes remains unchanged; only the annotation to the five sides of the pentagon was modified.



Figure 2. The most recent Singapore mathematics curriculum framework [9]

With the centrality of problem solving in the curriculum, the role of the calculator can thus be clearly articulated as playing a supporting role in facilitating the teaching and learning of mathematical problem solving.

2.1.2. Calculators and problem solving

In addition to being a tool for checking manual computation and an enabler for computations to be done by a machine, calculators can also play a critical role in mathematical problem solving. Researchers and teachers have recognized the calculator as being able to reinforce recognition of number patterns and handling computations involving larger numbers [4], [10], [11] and in [10], [11], who advocated the importance of explicitly exposing students to problem solving strategies, they also argued that the use of calculators reduced students' computational errors, thereby enhancing their success in problem solving. Researchers have also argued that the use of calculators is not necessarily restricted to reducing computational errors during problem solving; its use among mathematics students has resulted in them being more goal-driven, especially the high-scoring students, e.g., [12].

Efforts to enact problem solving in the Singapore mathematics classrooms at the secondary and pre-university levels have been ongoing in various research projects in Singapore, e.g., [13]-[15]. Such effort involves making problem solving heuristics and strategies the foreground of the courses, in other words, making problem solving explicit. In such problem-solving courses, as illustrated by the set of problems used for the course (http://math.nie.edu.sg/mprose in the Sample Problems section), the researchers have permitted the use of calculators to look for patterns and make conjectures. In other words, the role of the calculators took away tedious mundane computation from the students so that they could focus on pattern recognition and conjecturing, among other heuristics.

Some researchers (e.g., [6]) asserted that early use of calculators might have undesirable effects on students [16] and believed that students at the elementary grade levels should not be required to use

calculators, as they need to achieve proficiency in basic mathematical computations (e.g., perform the four operations without the use of calculators) and gain an essential understanding of mathematical concepts in order to move to higher levels. Calculators should only be allowed after they have achieved mastery of fundamental mathematics so that the role of calculators is then to reduce tedious and repetitive computation. This should occur at the upper elementary levels, where the role of calculators is to facilitate the students to explore relationships [17], that is, part of mathematical problem solving.

Calculators were permitted to be used at the primary five-level (students of age 11) in 2008 and at the primary six level (students of age 12) in 2009 in Singapore schools. This also resulted in calculators being permitted to be used at the national level examinations in one part of the paper, while the other part of the paper still prohibits the use of calculators in order to check students' competency with manual calculation. This new move was a reflection of the recognition of the power of the calculator while retaining the emphasis on building a strong foundation in students' computational skills.

Singapore teachers have ridden on the affordance provided by calculators to engage their students in mathematical problem solving at the primary school level. We shall illustrate with one example of such a task crafted for primary six students. Consider the following problem which was originally introduced for a problem solving course for lower secondary level (Leong et al., 2013)

Calculate the value of $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{99 \times 100}$

The formal solution to this question involves the use of summation of series using partial fractions, an advanced mathematical content that is not usually accessible to primary school students. With the use of a simple handheld or scientific calculator, the objective of this question has been converted to a task that reinforces the use of problem-solving heuristics of looking for patterns and making conjectures. A problem-specific scaffold that guides students towards developing problem-solving heuristics has been developed for this question appears below (Figure 3).



Figure 3. A scaffold for solving a problem on the summation of a series

through problem-solving with the aid of a calculator

In the above task, the use of calculators offloads the tedious calculation (including summation of fractions consisting of non-simplified terms) from the students. Note that calculation involving the addition of fractions containing the product of terms in their denominators could pose a challenge for most primary school students. This offloading of tedious calculations enables the students to focus on the prompts to look for patterns and eventually make conjectures for the last case involving a very large number of terms.

2.1.2. Graphing calculators at the pre-university level

The graphing calculator was permitted by the Singapore Ministry of Education at the preuniversity level, including its use as a pedagogical tool and in the final examination. The rationale of introducing a graphing calculator at the pre-university level was also analogous to that of allowing the use of the scientific calculator at the primary and secondary levels, in that it offloads students' repetitive plotting of graphs so as to switch the students' focus on the properties of the graphs of functions. Note that the graphing calculator was only introduced at the pre-university level so that students could have the opportunity to build up their competency in curve sketching at the secondary level before they start to use graphing calculators. However, it was still emphasized in the syllabus document that students need to be able to perform essential work rather than reading off answers from a graphing calculator (GC), although making effective use of such a tool is expected of students:

Where unsupported answers from GC are not allowed, candidates are required to present the mathematical steps using mathematical notations and not calculator commands [18, p. 2].

With the introduction of a GC, opportunities to explore various mathematical concepts involving graphs of functions also increased. This led to the feasibility of explaining advanced mathematical concepts through graphical means. For instance, the opportunity to develop an alternative understanding of the sophisticated concept of differentiability in calculus by linear localization, e.g., [19], was made possible in addition to the traditional method of using the secant line approach. Through local linearization, one is able to identify points on the graph of a function that is differentiable or otherwise: if one zoomzoom in sufficiently, the graph appears as a "linear function" as a function. This made it feasible to explain to high school students the notion of differentiability without delving into the formal definition of differentiation and differentiability.

Exploratory activities by means of a GC or other similar graphing tools have made it possible for students to explore graphs that are otherwise difficult to plot manually. Two examples of graphs which are usually not introduced without technology, but became a rich source of exploration due to graphing tools are shown in Figures 4 and 5.





Figure 4. The graph of the function $y = sin \frac{1}{x}$ is highly oscillating around x = 0

Figure 5. The graph of the function $y = x \sin \frac{1}{x}$ is also highly oscillating around x = 0, but continuous at x = 0. The properties of the graph can best be explored by using a GC with a zoom-in function

As illustrated above, GC and other graphing tools have ignited hope for students to explore in greater depth advanced mathematical concepts without prematurely being introduced to advanced mathematical definitions. Such tools could potentially widen students' exposure to a wider variety of examples from the domain of more advanced mathematics. It could further provide similar learning opportunities for both pre-service and practicing teachers who might be lacking in more advanced content knowledge such as calculus, e.g., [20], [21].

2.2. Phase two-technology and the mode of instruction

2.2.1. Reconceptualise the teaching of mathematics

Advances in the development of technology have led educators to question and review the traditional approaches to teaching and learning, e.g., [22], [23]. It has also challenged the traditional approaches and paradigms of mathematics and even the philosophy of mathematics instruction. As an illustration, geometry in the school mathematics curriculum was intended to "cultivate geometrical thinking" [24], focusing on Euclidean geometry theory from scratch using the most fundamental postulates and developing the theory thereby. Coxeter and Greitzer, among most mathematicians, asserted geometry as an ideal vehicle to introduce students to "axiomatics" because of its "esthetic appeal" [25]. Ayalon & Even, and Herbst recognize geometry as an ideal vehicle to develop deductive reasoning [26], [27]. Such a traditional approach has made school geometry inaccessible to many students.

Technology has led to a re-think of the traditional approaches to teaching geometry, in particular, with the use of dynamic geometry software. It has been reported that an intuitive-experimentalrigorous approach to teaching geometry has been observed in the Singaporean mathematics classroom. The sequence of teaching geometry no longer begins with definitions and theorems but begins with opportunity (afforded by technology) for students to explore and discover, understand, and apply geometrical properties. This is then followed by deductive reasoning (in solving a typical geometry problem) and eventually the formalism of mathematical proof. A detailed description of such an approach is described in [28]. Throughout this entire process of instruction enhanced by dynamic geometry software, the teacher used lots of visual images to aid students in visualizing the concepts and properties. As an example of this approach to teaching geometry, consider an example of a task that is used in a typical mathematics classroom (Figure 6).



Figure 6. A typical exploratory discovery worksheet accompanying the use of a Dynamic Geometry software

As illustrated in the above task, students are engaged in using Dynamic Geometry Software (DGS) to perform the above construction. Through multiple click-and-drag and the scaffolding provided in the worksheet, the students are led to discover the properties. The teacher next builds on the student's curiosity to introduce the notion of deductive thinking of mathematical proofs, followed by the formalism of geometry proof.

2.2.2. Mode of instruction

In recent years, especially following the lockdown due to the COVID-19 pandemic, videoconferencing tools such as Zoom and Teams have been rapidly adopted worldwide to combat the impossibility of face-to-face classroom instructions. This requirement for rapid adoption and familiarization has posed many challenges to teachers [29]. It could be tempting to rely almost exclusively on technology as the mode of delivery by "outsourcing the mathematical processes". This could have a negative impact on students' learning if they become reliant on technology without understanding the processes, e.g., [30], [31].

However, much progress has been made by the educators worldwide. The widespread use of video-conferencing tools has led to creative ways of instruction with the adoption of online and hybrid modes of teaching. The pandemic has challenged the age-old belief that in-person teaching is essential for good mathematics instruction [32]. It was interesting to note that the use of such video-conferencing tools had retained online instruction, both synchronous and asynchronous forms or a hybrid version, while methods to creatively engage students in active learning surfaced. Active learning is essential for developing and retaining a deep understanding of mathematics, e.g., [33]-[35].

Exemplars of mathematics lessons with strategies engaging engage students in active learning are beginning to appear in literature, e.g., [36], [37]. As an example, the use of the feature of breakout rooms in Zoom has made it possible to enact cooperative learning strategies; teachers have also explored other strategies such as think-pair-share online. It is not an exaggeration to say that the surprising gain due to the COVID-19 lockdown is the discovery of alternative modes of instruction that challenge the traditional belief of face-to-face instruction as the only ideal mathematics pedagogy. Such gain will be there to stay despite the easing of the pandemic. This new mode of instruction with the help of technology could indeed benefit distance learning in other parts of the world.

2.1.1. Flipped learning

Linked to the use of technology, the concept of "flipping classrooms" appeared. Soon after that, flipped classrooms and flipped learning became commonly discussed issues in Education. Typically, a flipped classroom is one in which the traditional exposition is done before class time, and more targeted student engagement is conducted during the in-class time, e.g., [37], [38]. The flipped classroom should ideally result in direct instruction being moved "to individual learning space, and the resulting group space is transformed into a dynamic, interactive learning environment where the educator guides students..." [39, para. 1]. Such a form of learning is termed "flipped learning". Note that the terms "flipped classroom" and "flipped learning" might not be synonymous. Researchers have cautioned that flipped classrooms might not result in flipped learning, e.g., [40].

There does not exist a unique approach to flipped classrooms. However, the flipped classroom is usually characterized by the greater use of online learning rather than textbooks. Not only that, teachers are likely to spend more time facilitating learning during the class rather than on whole-class instruction [41]. With such features, the flipped learning approach is likely to result in engaging students who are likely to find greater satisfaction with homework tasks. It should be noted that currently empirical evidence of the effect of flipped learning on students' academic performance might not be well established currently, e.g., [42], [43]. However, some researchers believe that the flipped learning approach in mathematics, based on the general guiding principles, will likely have the potential for differentiated instruction [40].

Flipped learning has been adopted at various levels in the Singapore education system, including a mathematics course in teacher education course [34]. Here we report on an innovative effort conducted at the secondary level for low-achieving students. The content of the course we developed for the low achieving students has been documented in [44]. Here we focus more on the flipped section of the course content.

The course that we developed involved two topics in lower secondary school mathematics: Percentage and Statistics. We used comics to develop a "replacement unit" [45] for the content in replacing the usual textbooks and worksheets. Students, especially the low-achieving and unmotivated students, and teachers have long realized the usefulness of comics and other elements of pop culture in teaching mathematics [44], [46]. Our development of the comics replacement units was in response to the students' (and teachers') wishes. We developed a full online package and a hardcopy package on these two topics. We proposed that the package (both the online and hardcopy versions) be read by the students prior to the face-to-face lessons, while teachers could focus on the essence of mathematics during the lesson. We provided the teachers with proposals for enacting the classroom lessons while allowing the teachers to adapt modify (or even discard) our proposal.

In our study on the two topics Percentage and Probability, we witnessed encouraging signs from both students and teachers [47], [48]. It was observed that the lessons attended the lesson fully prepared in that they had read the comics passages. Consequently, the teachers were able to engage in activities with the students (including role play and authentic data collection exercises), thereby delving deeper into the mathematical content knowledge.

In the lessons we observed in [47], [48], it was evident that the students had already read the comics and completed some accompanying tasks (although they might not have completed all the tasks). During the lessons, the students asked the teachers questions about the comics (and the mathematics embedded in the comics), while the teachers engaged the students in various activities, and also challenged these students with higher-order questions. It was also interesting to observe at 24

certain junctures of such lessons, the students directed the pace of the lesson by clarifying or telling the storyline of the comics. Note that, however, not all the participating schools adopted the online versions of the comics provided by the researchers. Several schools printed out hard copies of the comics for the students prior to the lessons.

2.3. Phase three - Technology and the mode of instruction

While the first two phases described above challenge our traditional belief about the role of teachers and the learning environment in education, the current phase of the advent of the GAI can be said to revolutionize the entire education with radical changes to the entire educational landscape. This is especially true with the recent advent of the ground-breaking AI applications, in particular, ChatGPT. Even though its appearance was a rather recent event in November 2022 [49], a search in the ERIC database with the words ChatGPT and GAI shows more than 100 journal articles published on this topic, demonstrating the abundant attention that GAI applications are receiving.

ChatGPT is a "powerful machine learning software that uses the Generative Pre-trained Transformer (GPT) algorithm to generate human-like responses to text-based inputs" [50]. Training on a large corpus of data and through a process of dialogue optimization, ChatGPT can respond to prompts in a conversational manner much like a human being [51]. ChatGPT has been able to perform language translation, text summarization, creative writing and even writing and debugging programming codes [52], and even pass university examinations, e.g., [53]. Educators are currently exploring the challenges and opportunities that the open AI tool brings to education, and how in particular to harness its power in education, [54]-[57]. The academic integrity aspect of its use in education is also a crucial point of discussion, e.g., [58]. For example, [57] cautioned against a potential problem of "cheating in assessment", perhaps also a challenge to re-think an appropriate mode of assessment in the error of GAI. Despite much diverse discussion, generally, educators are positive about its advent, while recognizing its challenges. At the current stage, not much has been explored on the use of ChatGPT or its impact on mathematics instruction.

In this section, I present my use of ChatGPT in relation to mathematics problems and the implications that can be obtained from this interaction. One of the problems with ChatGPT is perhaps its ability to answer most questions as if these answers are provided by a human expert. However, ChatGPT is also capable of hallucination, giving incorrect answers while presenting the answer in a professional form. Consider the following examples that I obtained in preparing this paper. I labelled what I typed in verbatim as "PROMPT" and ChatGPT's responses as "CHATGPT". It should be noted that ChatGPT's response might not be replicable; the same prompt typed in might not entail the same response from ChatGPT. This is understandable as ChatGPT makes use of machine learning on a rapidly increasing data set, and the optimization involves some randomness in providing the answer.

PROMPT1:>	Solve the equation $3x - 2 = x + 5$.
CHATGPT:>	Sure! To solve the equation $3x - 2 = x + 5$, you can start by isolating the variable
	x. Here are the steps:
	Subtract x from both sides:
	3x - 2 - x = x + 5 - x
	Simplify the left side:
	2x - 2 = 5.
	Add 2 to both sides:
	2x - 2 + 2 = 5 + 2
	Simplify both sides:
	2x = 7
	Divide by 2:
	2x/2 = 7/2

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	Simplify:
	$x = \frac{7}{2}$.
	So, the solution to the equation $3x - 2 = x + 5$ is $x = \frac{7}{2}$.
PROMPT2:>	Solve the equation $(2x-1)/(x+1)=5$.
CHATGPT:>	Certainly! Let's solve the equation. $\frac{2x-1}{x+1} = 5$.
	First, we can cross-multiply to get rid of the fraction:
	2x - 1 = 5(x + 1).
	Now, distribute 5 on the right side:
	2x - 1 = 5x + 5.
	Next, move all the <i>x</i> terms to one side and the constants to the other:
	2x - 5x = 5 + 1
	Simplify:
	-3x = 6
	Now, divide by -3 to solve for <i>x</i> :
	x = -2.
	So, the solution to the equation is $x = -2$.

Note that ChatGPT was able to give correct solutions to the typical problems on solving a linear algebraic equation and a fractional equation. A fairly detailed explanation of the solution was also provided, although it could be argued that the two solutions above make use of different paradigms of solving an equation: the first one involving balancing two sides of the equation and the second on shifting terms between left and right side with a change in operations. In the responses to the above two prompts, ChatGPT seems to be able to perform the explanation job as efficiently as a human teacher! I next extended the problem of solving a linear algebraic equation further to a generalized form below as a prompt for ChatGPT below.

PROMPT3:>	Solve the equation $ax + b = cx + d$ for x in terms of a, b, c and d.
CHATGPT:>	To solve the equation $ax + b = cx + d$ for x, you want to isolate x on one side of
	the equation. Here's how you can do that:
	ax + b = cx + d
	Subtract <i>cx</i> from both sides:
	ax - cx + b = d
	Factor out <i>x</i> from the terms on the left side:
	x(a-c)+b=d.
	Now, subtract <i>b</i> from both sides to isolate the term with <i>x</i> :
	x(a - c) = d - b.
	Finally, divide both sides by $a - c$ to solve for x:
	$x = \frac{d-b}{d-b}$
	a-c
	So, the solution to the equation $ax + b = cx + d$ for x is $x = \frac{a}{a-c}$.
PROMPT4:>	So how many solutions does the equation $ax + b = cx + d$ have?
CHATGPT:>	The equation $ax + b = cx + d$ has a solution for x as long as the denominator $a - b$
	c in the expression $\frac{d-b}{q-c}$ is not equal to zero.
	Therefore, if $a - c \neq 0$, There is a unique solution for x, and the equation has
	exactly one solution.
	If $a - c = 0$, the equation will have no solution if $d - b \neq 0$ (inconsistent
	system) and infinitely many solutions if $b = d$ (dependent system).
	In summary:

- If $a - c = 0$ and $d - b \neq 0$, the equation has no solution.		
	on.	
- If $a - c = 0$ and $d = b$, the equation has infinitely many solutions.	solutions.	

When ChatGPT is asked to solve the generalized linear equation in PROMPT3, a general formula for the solution is provided without considering various possible values of the numbers or distinguishing between different cases; such distinguishing cases occur only when PROMPT4 is used. Following this prompt, a complete solution with a detailed explanation distinguishing between all the possible cases was provided. When asked deeper into the geometrical interpretation of solving a linear equation as in PROMPT5, a clear explanation of the various cases of pictorial representation was provided, most likely having learned from PROMPT4.

PROMPT5:>	Can you give me a geometrical interpretation of solving $ax + b = cx + d$?
CHATGPT:>	The equation $ax + b = cx + d$ can be interpreted geometrically in terms of lines on the
	coordinate plane.
	Consider the equation as $y = ax + b$ and $y = cx + d$ where y represents the vertical
	coordinate and x the horizontal coordinate.
	The equation $y = ax + b$ represents a line with slope <i>a</i> and y-intercept <i>b</i> . Similarly, $y =$
	cx + d represents a line with slope c and y-intercept d.
	The solution to the equation $ax + b = cx + d$ is the x-coordinate at which these two
	lines intersect. Geometrically, it's the point where the two lines coincide on the
	coordinate plane.
	- If the lines have different slopes $a \neq c$, they will intersect at a unique point, and the
	system has a unique solution.
	- If the lines have the same slope $a = c$ and different y-intercepts $(b \neq d)$, the lines are
	parallel and do not intersect, indicating no solution to the system.
	- If the lines have the same slope $a = c$ and the same y-intercept $b = d$, the lines
	overlap, and there are infinitely many solutions to the system.
	So, the geometric interpretation involves considering the intersection (or lack thereof)
	of lines on the coordinate plane.

Based on the above illustration, it appears that the user of ChatGPT needs to "ask the correct questions" by entering relevant prompts to elicit desirable answers to a given problem. We next illustrate with another example of solving a quadratic equation, using two different methods.

PROMPT6:>	Solve the equation $x^2 + 3x - 1 = 0$, give your solution in exact form.
CHATGPT:>	To solve the quadratic equation $x^2 + 3x - 1 = 0$, we can use the quadratic formula:
	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ this equation, <i>a</i> , <i>b</i> , and <i>c</i> are the coefficients of the quadratic equation
	$ax^2 + bx + c = 0$
	For the given equation $x^2 + 3x - 1 = 0$, the coefficients are:
	a = 1
	b = 3
	<i>c</i> = -1
	Now, plug these values into the quadratic formula:
	$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-1)}}{2(1)}$ Simplify the expression under the square root:
	$x = \frac{-3 \pm \sqrt{9+4}}{2}$
	$x = \frac{-3 \pm \sqrt{13}}{2}$

	So, the solutions to the equation $x^2 + 3x - 1 = 0$ in exact form are:
	$-3+\sqrt{13}$
	$x = \frac{1}{2}$
	$r = \frac{-3 - \sqrt{13}}{12}$
DDOMDT7.	Any you able to colve the above equation by taid and amon factorization?
PROMP17.>	Are you able to solve the above equation by that-and-error factorization?
CHATGPT:>	Certainly! To solve the quadratic equation $x^2 + 3x - 1 = 0$ by trial-and-error
	factorization, we need to find two numbers whose product is the product of the
	coefficient of x^2 (which is 1) and the constant term (which is -1), and whose sum is the
	coefficient of x (which is 3). (*)
	The product is $1 \ge -1$, and we are looking for two numbers whose sum is 3.
	The numbers that satisfy these conditions are 4 and -1 because $4 \ge (-1) = -4$ and $4 + (-1) = -4$
	1) = 3.
	Now, we can express the middle term $3x$ using these two numbers:
	$x^2 + 4x - x - 1 = 0.$
	Now, group the terms:
	$(x^2 + 4x) - (x + 1) = 0.$
	Factor by grouping:
	x(x + 4) - (x + 1) = 0.
	Now, factor out the common factors:
	x(x+4) - (x+1) = 0.
	The factored form is $(x - 1)(x + 4) = 0$. (**)
	Now, set each factor equal to zero and solve for x:
	x-1=0 implies $x=1$ and $x+4=0$ implies $x=-4$.
	So, the solutions to the quadratic equation $x^2 + 3x - 1 = 0$ by trial-and-error
	factorization are $x = 1$ and $x = -4$.

In solving a quadratic equation, it seems the response to PROMPT6 to solve the quadratic equation, ChatGPT first uses the quadratic formula method. Also, it was able to recognize an alternative method (trial-and-error factorization) method of solving a quadratic equation under PROMPT7, it appears that at this juncture ChatGPT was unable to synthesize the entire procedure of factorization by trial-and-error as illustrated by the incomplete argument in (*) and an erroneous factorization in (**). Thus, the user of ChatGPT must not only know how to use appropriate prompts to illicit appropriate responses from ChatGPT but must also detect errors in arguments and computations. We further illustrate by one more example of using ChatGPT to prove a result in geometry.

PROMPT8:>	Prove that the two base angles of an isosceles triangle are equal.
CHATGPT:>	Let's consider an isosceles triangle, where two sides are of equal length. Without loss
	of generality, let's label the sides as $AB = AC$.
	Now, we want to prove that the two base angles, $\angle ABC$ and $\angle ACB$, are equal.
	Consider the isosceles triangle ABC. Since $AB = AC$, the triangle has two equal sides.
	Let's denote the third side as BC.
	Now, draw the altitude from vertex A to the base BC, and let's call the point of
	intersection D. This altitude divides the isosceles triangle into two right-angled
	triangles, triangle ABD and triangle ACD.
	In triangle ABD:
	-AB = AC (given, as the triangle is isosceles).
	- $\angle ADB = 90^{\circ}$ (by definition of altitude).
	In triangle ACD:

	-AC = AB (given, as the triangle is isosceles).
	- $\angle ADC = 90^{\circ}$ (by definition of altitude).
	Now, since AB = AC and $\angle ADB = \angle ADC = 90^\circ$, we can conclude that triangle ABD
	and triangle ACD are congruent right-angled triangles by the Hypotenuse-Leg
	Congruence criterion. Therefore, the corresponding parts of congruent triangles are
	equal. In particular, $\angle ABC = \angle ACB$ because they are corresponding angles in
	congruent triangles.
PROMPT9:>	Can you prove the above result without drawing any additional lines?
CHATGPT:>	Consider an isosceles triangle ABC where $AB = AC$. We want to show that $\angle ABC =$
	∠ACB.
	1. **Statement:**
	$\angle ABC = \angle ACB.$
	2. **Proof:**
	In triangle ABC, since $AB = AC$ (given that it's an isosceles triangle), we can say that
	the two sides opposite the base angles are equal.
	By the Isosceles Triangle Theorem, the angles opposite those equal sides are also
	equal.
	Therefore, $\angle ABC = \angle ACB$.
	So, without drawing any additional lines, we have shown that the two base angles of
	an isosceles triangle are equal.
PROMPT10:>	By the way, what is the Isosceles Triangle Theorem?
CHATGPT:>	The Isosceles Triangle Theorem states that in an isosceles triangle, the angles opposite
	the equal sides are also equal. In other words, if two sides of a triangle are congruent
	(have the same length), then the angles opposite those sides are also congruent (have
	the same measure).
	Mathematically, if ABC is an isosceles triangle with $AB = AC$, then $\angle ABC = \angle ACB$.
	Conversely, if $\angle ABC = \angle ACB$ in triangle ABC, then $AB = AC$.
	This theorem is useful when working with isosceles triangles, as it allows you to relate
	the measures of angles in the triangle to the lengths of its sides.

ChatGPT gave a correct proof of a geometry result (a standard result with a standard proof) in PROMPT8, it ended up with a circular argument when asked to offer an alternative proof of the theorem in PROMPT9. PROMPT10 was to confirm the result used by ChatGPT was a consequence of its hallucination or learning from an incorrect data set.

Learning from the above interplay with ChatGPT, it appears that much procedural mathematics can be obtained from ChatGPT directly. Thus, if learning mathematics is merely restricted to the acquisition of procedures, it seems plausible that ChatGPT can fulfill the job most of the time. However, despite the professional outlook of the response by ChatGPT, the mathematical content might not be correct all the time. In short, the user must be able to ask the right questions and detect errors in the responses provided by ChatGPT. The advent of ChatGPT challenges the traditional work of the teachers and students: ChatGPT can easily complete all homework assignments of the teachers. Teachers cannot possibly fight the battle against such emerging GAI but to think of the best ways to engage these for students' learning. Issues such as integrity and the appropriate mode of assessment in such an era need to be deliberated by educators and policymakers.

3. Conclusions

In the above discussion, it is clear that each phase of new technology that was introduced into the curriculum appears to challenge the existing paradigm of education. When the calculator was first introduced, it challenged the importance of mental calculations versus reliance on the tool for calculators. The traditional teaching paradigm was challenged, but it was reconciled with an emphasis on building students' higher-order thinking skills over repetitive mundane calculations. The advent of computer software to aid the teaching of mathematics, challenges the importance of rigor of mathematics over "intuitive" approaches; still, it was reconciled by emphasizing the equal importance of relational understanding over instrumental understanding.

The introduction of online platforms such as Teams and Zoom, and the concepts of flipped learning enable the process of learning not to be restricted to the traditional classroom; learning can take place in different parts of the world. This phase has allowed education to continue even during the Covid-19 lockdown in which face-to-face lessons were banned in different parts of the world. Educators need to re-interpret the role of traditional classrooms in the place of education when even online platforms allow interactive activities among children.

The most recent advent of GAI such as ChatGPT questions the content of education; as illustrated in this paper, most of the procedural mathematics tasks can be managed by ChatGPT, and also detailed explanations can be provided. Thus, ChatGPT not only functions as a provider of content knowledge but also as a teacher with a detailed explanation of the procedures. Although it was illustrated above that there were some errors detected, these errors will soon be overcome with the increasing data set that ChatGPT is exposed to every instance.

To face the rapidly changing educational landscape due to technology, teachers must be prepared to manage the technological tools. Instead of fighting a losing battle in resisting technology, teachers could be prepared for how best to harness them in teaching mathematics. Another word of caution is that such rapid change in technology will most likely result in equity issues and could potentially widen the divide in society. This could be a challenge to most educators and policymakers.

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