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FOUR PERSPECTIVES ON TWO FUNDAMENTAL CONCEPTS - MATHEMATICS AND PHYSICS TEACHERS' PERCEPTIONS OF MODELS AND MODELING IN THE CLASSROOMS

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Abstract. The terms model and modeling are central concepts in mathematics and physics teaching. It is known, however, that the focus of the perspectives on these concepts in the two subjects are shifted against each other. Therefore, as part of an interview study, the views of teachers who teach both mathematics and physics are investigated. The article presents the results of this study for the concept of modeling, for which no fundamental differences can be found, but attitudes that are not conducive to successful integration in the classroom. Results on the different perceptions of central concepts in questions of interdisciplinary teaching and also in increasingly widespread STEM lessons appear to be particularly significant. Finally, selected impulses for the integration of modeling tasks are briefly outlined.

Keywords: models, modeling, mathematics education, physics education, teachers' perspectives.

1. Introduction

The terms model and modeling are both found in mathematics and physics lessons and are important topics. It is also known that disciplines can have different views on both concepts [1]. This raises the question of how the concepts are viewed in interdisciplinary contexts, e.g., in STEM lessons. The perspective on STEM education appears to be significant, especially due to the global trend of integrating natural science subjects with mathematics education [2]. At the same time, the more limited perspective on mathematics and physics education is significant too, as there is a very close connection between these two subjects, manifesting itself in conceptual and historical ways as well as in the modes of representation [3], [4].

The educational system in Germany can provide initial insights into this question, as teachers have always been trained in two subjects in this country. The combination of physics and mathematics is very common. The empirical study on which this article is based therefore examines the views on the concept of modeling as it is relevant for teachers with the subject combination of mathematics and physics at secondary schools in Germany. The authors aim to identify possible commonalities, differences, or even inconsistencies in conceptual views as well as in practical implementation in teaching. It is also significant to compare attitudes and expectations with the known benefits of incorporating modeling tasks in education from the literature, as it can reveal motivational potential.

From previous studies, it is already known that the focus of teaching about models and modeling in both subjects is shifted relative to each other [1] - Mathematics education seems to focus more on the process of modeling while physics education has a stronger focus on the models as a product of this process. The relevant educational standards provide an initial indication of the orientation of the subjects. However, these standards also make clear that there is no restriction of the two subjects to one of the respective terms.

2. Content

2.1. Models and modeling in mathematics and physics education

2.1. Models and modeling in the educational standards

Initially, the significance of the terms “model” and “modeling” in the disciplines of mathematics and physics will be briefly outlined. Since the empirical part of the study involved surveying teachers from Germany, at this point, the educational standards in Germany will also be taken into account [5], [6]. In both cases, we limit ourselves to the standards for the upper level of secondary education (class 11 to 13). Regarding the subject of physics, the educational standards state that it is a theory-guided empirical science. Physics allows processes beyond human perception to be describable through models. The standards further specify that learners should recognize the significance of abstract, idealized, and formalized descriptions through mathematical modeling and predictions. At the same time, it is emphasized that the limited validity of existing models should also be taught. It becomes clear that both the model itself and the process of modeling are mentioned as components of teaching. However, the emphasis is slightly shifted towards the models.

For mathematics, the standards call for the treatment of transitions from real situations to mathematical concepts. It is explicitly emphasized that this includes both the construction of mathematical models and the understanding and evaluation of existing models. In addition to the creation of models based on given real situations, there is also a demand for the reference of the results of mathematical modeling back to these real situations. For the educational standards in mathematics, the situation is reversed. Although both terms are mentioned here as well, the focus is quite clearly on modeling.

2.1.2. Literature review on the concept of modeling

The international didactic literature on the topics of models and modeling can be traced back quite far into the past. For instance, Schlichting [7] identified the two poles of constructive model construction and reproductive model application. In his view, independent model construction by students is particularly suitable for providing insights into the genesis of knowledge that goes beyond the known historical approaches. He also emphasized the importance of creative elements within the process and called for the explicit naming of the modeling process itself and its subsequent reflection. As an application example, he mentions black-box experiments, as known from electrodynamics, optics, or mechanics. In such experiments, models are to be created for groups of components that are hidden in a box. The investigation methods are limited to observations and non-invasive measurements, such as the recording of current-voltage curves at the outer contacts of electrical circuits.

In current literature, modeling is mostly seen as the identification of relevant elements and the discovery of related quantities and laws [8]. However, from a physical perspective, the application of mathematical methods is only one option. The focus can be also more on conceptual explanations of physical models rather than quantitative aspects.

For mathematics education, the understanding of the relationship between mathematics and the real world is often mentioned as a key goal. Typically, such an interconnection between a model and the real world is represented through so-called modeling cycles [9], [10]. The incorporation connections of to the real world into mathematics education is frequently justified in Germany through the mathematical basic experiences (in German: Mathematische Grunderfahrungen) according to Winter [11]. These basic experiences demand, among other things, "to perceive and understand phenomena of the world around us, [...] from nature, society, and culture in a specific way" and "to acquire problem-solving skills (heuristic skills) that go beyond Mathematics in dealing with tasks" (translation by the authors). However, it should be also noted that the products of such modeling processes often consist merely of numbers, which are later given a unit and then transferred back to the real situation [8].

Even in terms of specific, shared applications, differences in perspectives and approaches to modeling in mathematics and physics education become apparent. For instance, when dealing with 3D printers, the physics class predominantly focuses on handling models created by the teacher. In contrast, examples from mathematics lessons show that a process-oriented approach is followed in the majority of cases, i.e. the focus is on planning and creating models [1].

Regarding the functions that modeling can fulfill within the context of teaching, reference can be made to the functions of using models according to Kircher [12]. The functions mentioned there, such as explaining, predicting, and learning through models, can also be transferred to the process of modeling. Without going into further detail here, there is no particular priority assigned to these functions. Making predictions through modeling is not more important in school contexts than explaining how a system works or understanding an issue through modeling. This statement can be understood on the basis of the educational standards as well as the literature and the results of the empirical part of the study.

While it is predominantly observed in the literature that models and modeling are conceptually treated separately, this is not always the case. For example, Oh & Oh discuss that "one of the tenets of using models in science education is students' active participation in diverse modeling activities" [13, p. 1122]. While it is possible to subsume one term under the other, we want to take a different approach here and aim for a clear conceptual separation. This appears to be a justified approach both in advance, regarding the initial situation described in the literature, as well as following the evaluation of the interviews conducted.

2.1.3. Initial situation for the empirical study

The starting point before conducting the empirical study can be summarized based on the previous discussions as follows:

- The concepts of the model and modeling are significant topics in both physics and mathematics education.

- Initial differences are found in the emphasis on dealing with models and modeling in the two subjects.

- There is no general definition for both concepts [14].

We align with the view of Oh and Oh [13], who states that "[...] teachers of science need to understand the nature of models and modeling in science more clearly and reflect this understanding in their science instructions." At the same time, we see an extension of this demand as necessary for the group of mathematics teachers.

2.2. Research question and method

2.2.1. Research question and method

Based on the need for an understanding of these concepts, we aim to capture the perspectives of selected mathematics and physics teachers on both concepts as an example. The overarching research question is: What beliefs do teachers of mathematics and physics have about models and modeling in the classroom?

We use the term "belief" following Pehkonen and Pietilä [15, p. 2]:

"An individual's beliefs are understood as his subjective, experience-based, often implicit knowledge and emotions on some matter or state of art. […] Beliefs represent some kind of tacit knowledge. Every individual has his own tacit knowledge, which is connected with learning and teaching situations, but which rarely will be made public."

The challenge, therefore, is to encourage teachers to express their beliefs explicitly or at least implicitly by using examples from their own teaching. For this reason, the empirical investigation will be conducted in the form of semi-structured interviews. The interviews are divided into three parts:

1-Experiences and beliefs regarding Physics education

2-Experiences and beliefs regarding Mathematics education

3-Integration and summary of perspectives on both subjects

For further clarification, the teachers are asked to illustrate their explanations with examples of concrete situations from their own lessons.

2.2.2. Research method

The interviews were conducted in German in order to avoid distortions in the descriptions and paraphrases through translations. The interview material was transcribed and then analyzed using a structured content analysis according to Mayring [16]. Afterwards, the results were translated into English.

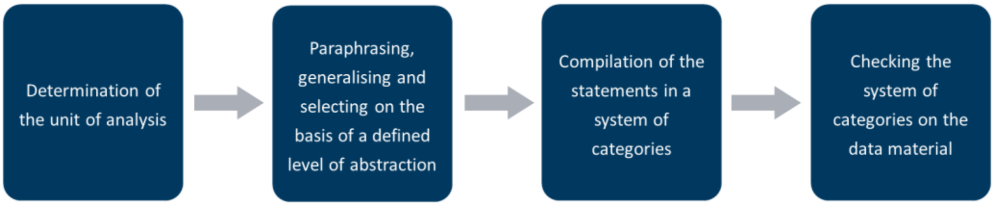

Figure 1. The sequence of the qualitative content analysis according to Mayring

The assignment of the teacher's statements to the categories was based on an independent analysis by the authors followed by discussion and consensus when there were differing classifications. There was no need for a subsequent change in the category system after the analysis of the material.

The interview material provides extensive insights into teachers' attitudes towards models and modeling, as well as their individual practices related to these concepts in teaching. Due to space constraints, we limit ourselves here to presenting partial results on the topic of modeling in physics and mathematics education. The relevant sub-questions from the interviews are the following:

- Do the students create their own models in mathematics/physics? If yes, how does the process work?

- Why should we model in physics/mathematics classes?

The material was analyzed in its entirety, meaning all passages where the process of modeling was mentioned were used for this evaluation.

2.3. Results of the empirical study

Based on the interview material, the category system was developed inductively. Four main categories emerged within this analysis (the main categories and subcategories shown below represent the results of step 4 of Mayring’s structured content analysis) as follows:

- Reasons for modeling
- Prerequisites, conditions for success and obstacles
- Nature of the model in the process of modeling
- Working with the model as a product

The category “reasons for modeling” arises directly from the corresponding sub-question in the interview guidelines, in which such reasons are asked. The question itself is not further restricted, i.e., there were no expectations regarding answers relating more to pedagogical (e.g., understanding) or other reasons (e.g., nature of science).

Under the category “prerequisites, conditions for success and obstacles”, those answers were subsumed which concern both the conditions for successful modeling as well as for the failure – or rather for the non-implementation of modeling tasks in the classroom.

The “nature of the model in the process of modeling” refers to the physical or non-physical nature of the model within the modeling process. Particularly for this question, several attempts were made to gain more precise insights into the type of model by asking specific questions, as this was not always clear from the teacher’s initial statements.

The final category deals with models as the outcomes of modeling tasks (in contrast to the model as a finished product brought into the classroom) are covered by very few interview passages. However, it is a result that is clearly different from other categories, so that a corresponding main category could be derived from it.

In the following, the results are presented, and organized according to the four main categories. These main categories, as far as the material indicates, are subdivided into subcategories. The presentation is done separately for the subjects of mathematics and physics. To illustrate the categories and sub-categories, interview excerpts are quoted (translations provided by the authors), which are representative of the respective abstracted statements.

2.3.1. Reasons for modeling

** Reasons for modeling in physics education*

Regarding physics education, two subcategories related to the reasons for modeling are identified. The first category is called pedagogical-educational, focusing on improving the understanding of topics perceived as difficult. An example mentioned within this context is the creation of a model of the solar system, which various teachers talked about relatively frequently (all emphasis in this and all subsequent interview excerpts by the authors):

“Yes, simply so that the students have something, um, tangible. Things that are seemingly difficult for them, um, they can then understand much better. For example, with um, light and shadows, Earth, Moon, Sun. I always felt back then, um, that they understood it better”.

Another teacher argued in line with a Nature of Science approach at this point:

“[...] I'm back to the atomic models, um, if I want to understand the step and the expansion of these models, I can’t avoid modeling it myself first and then maybe really going through a process of knowledge acquisition”.

The process of knowledge generation, here using the example of classical atomic models, should be traced and understood by students entering a modeling process. Again, more explicitly towards the nature of physics, it is argued elsewhere:

“I don't think the last question [if modeling is done in the physics class] arises because you can't avoid it, so I believe, um, whether one should model, I think it happens automatically when you engage in physics.”

Engaging in physics, or rather physics education, is equated here with the formation of models about nature, comparable to the statements found in the educational standards about the nature of the subject and its connection to models and modeling.

**** Reasons for modeling in mathematics education***

With regard to mathematics education, unlike physics education, only reasons classified as educational-pedagogical are mentioned. Most frequently mentioned is student activity that arises when working on modeling tasks. The openness of such tasks, including their approaches and solutions, is also positively emphasized:

“[...] what student-activating means, you know, so students think concretely on their own, maybe with rules they've learned before, and then it's more or less about application, okay, how do I solve this problem now? Um, problem-solving is, um, the starting point. I have some sort of problem situation and okay, how do I approach it? That's much more meaningful and productive [...]”

2.3.2. Prerequisites, conditions for success and obstacles

In the second category, responses are summarized that relate to the prerequisites for successful modeling and possible obstacles. Regarding physics education, concerns about the feasibility of modeling tasks in the classroom are frequently expressed, which can be summarized in two subcategories: Organizational conditions and the competencies of the students. In a quote, all these concerns are particularly succinctly summarized:

“Yes, the time required and because, um, in physics, this quantitative aspect keeps diminishing, so we work more centered around phenomena, that's what it's about again, um, understanding again, how it fundamentally works, but the calculations come later, yes.”

In addition to general concerns regarding the available teaching time (physics is usually taught for a maximum of two hours per week in grades 5 to 10 in Germany), another particularity of physics education in Germany is emphasized here. The teaching, especially in non-high school settings, cannot predominantly rely on a quantitative approach but has to work phenomenon-oriented. Similarly, there is often mention of the perceived lack of creativity among students:

“Yes, that would naturally be, um, well, of course, the question is how creative the students are (laughs), exactly, it’s a great idea in itself, the question is just how to implement it in class with the time, um, if you tell them, well, you are now, one has to. Well, then I'm setting guidelines again.”

With the goal of genuine openness in mind, it is emphasized in this statement that the presumably necessary support and organizational conditions in the implementation hinder a genuine, process- and result-open modeling cycle.

For mathematics education, teachers see similar difficulties, with an initial advantage over modeling in physics education being mentioned. The transfer from everyday life to a mathematical model is considered to be easier than it should be for a physical phenomenon or experiment. However, the obstacles of lacking skills and the necessary creativity are also seen as hindrances in mathematics lessons. In addition, there is another problem with the lack of explicit consideration in the guidelines:

“[...] modeling tasks [...] there are just so many different ways to arrive at a solution, and yes, logically, that also happens in my math classes, but such specific modeling tasks that are labeled as such, that's rarely the case for us, um, at most, it's an extension option for the very fast or very good students [...]”

From the context of the interview, it becomes clear that this statement primarily refers to the mathematics textbook, which seems to have a strong guiding function on the teaching of the respective teacher. It is also evident overall that mathematics education, compared to the subject of physics, is subject to significantly greater limitations in design, as it must meet the requirements of central interim and final exams as well as comparative assessments, leaving less room for instructional excursions.

2.3.3. The nature of the model in the process of modeling

In this category, responses are summarized that describe the nature of the model (solely as the product of the modeling process) and its relationship to the original objects and concepts. For physics education, tangible and iconic models are mentioned, i.e., those that have been made or drawn by students themselves.

“How do I represent that in my model? And when it comes to, um, I'm doing something manually, I’m building my solar system out of papier-mâché, I’m making a decision there too. Is the size ratio between my planets important or not? Likewise, if I’m not doing something manually, but I’m drawing, I’m drawing a circuit, am I making any decisions there too?”

Almost overwhelmingly, with reference to physics education, tangible models are mentioned. Other forms of models, such as the drawn circuit mentioned in the preceding quote, are mentioned much less frequently. In accordance with the educational standards, elsewhere in the material, it is pointed out that the models are derived from observations, emphasizing the empirical aspect of physics. For mathematics education, a similar argument is made, highlighting that modeling involves a transfer from real situations to mathematics. At the same time, this is the only type of expression that concerns the nature of the model within mathematics education. Inner-mathematical modeling is not described at any point. On the contrary, the teachers make it clear that these newly created models, at least from the student’s perspective, are truly novel models. This once again emphasizes the openness of the process and the product.

2.3.4. Handling the model as a product

In the fourth main category, statements about dealing with the model, as the product of the modeling process, are subsumed. However, the number of statements that can be assigned to this category is very limited. In physics, the reference to a model of the solar system is found again, which is also intended to serve for understanding as a finished product:

“Yes, simply so that the students have something, um, more tangible, something difficult, which is presumably difficult for them, um, then they can understand it much better. [...] They have to figure it out for themselves, like do I use a big ball for the Earth, a small ball for the Moon, then they have to think about these things additionally.”

In contrast, there is no statement within the material on mathematics education about the further use of the results of a modeling process.

2.3.5. Summary of the empirical study

Overall, and in comparison to the statements about the use and handling of models in the teaching of both subjects, it can be observed that the teachers’ statements about modeling lag significantly behind. This applies to both the extent and the level of detail of the statements. This finding corresponds, among other things, to our own previous work, which identified a focus on the model itself [1].

Contrary to our expectations, however, is the observation that the statements about modeling in physics education are generally more extensive than those in mathematics education. This may, however, be attributed to the established sequence in which the subject of physics was discussed first, followed by the subject of mathematics.

Largely in line with expectations, but remarkable in the strength of expression, is the almost complete ignoring of models that have emerged as products of a modeling process. While there is at least one statement regarding a physics lesson where the resulting model is intended to serve as an aid to understanding, there is simply no mention of it in mathematics education. Fully circular processes, as known in the educational literature (e.g., from planning to production to the improvement of 3D models, see [17]), are not mentioned at any point. The same applies to theoretical models, by which we mean models without an empirical reference in mathematics or non-objective or iconic models in physics. These are not mentioned at all for mathematics education. For physics, there is an indirect mention when it comes to understanding historical ideas about atoms.

Overall, the teachers surveyed have a positive attitude towards modeling in mathematics and physics lessons. It is important to consider the inherent issue of the data collection method due to the social desirability of responses. The details of the responses then also contain some critical attitudes and comments. These include general doubts about the feasibility of modeling in physics lessons and the difficulties of modeling in secondary level I (grades 5 to 10). The latter point is often attributed to the predominantly qualitative way of working in these grades. The creation of tangible models, however, is viewed positively. The observed stronger focus on phenomena and specific models, according to Schlichting, does not contradict the fundamental idea of modeling. On the contrary, these conceptual steps are essential components of the process.

Especially with regard to the value of the conceptual working method, the following critical attitudes are to be seen as an unfavorable constellation for the integration of modeling in the classroom. Thus, in many interview excerpts, a critical attitude towards the necessary creativity became apparent, with the teachers doubting whether the pupils have adequate creativity at their disposal.

Overall, it can be said that, within the framework of Schlichting's approach, i.e., in the distinction between reproductive model use and constructive model finding, the teaching of the interviewed teachers takes place much more on the side of reproductive model use. The creation of novel models by the students themselves occurs only to a lesser extent, and when it is mentioned, at least the products of this process are not further used.

In view of the goals formulated in the educational standards and the educational literature, this situation must appear unsatisfactory, as independent modeling is already considered possible and useful below the high-school level: "Student involvement in modeling activities can enhance science learning in secondary and elementary schools as well" [13, p. 1122].

2.4. Suggestions for teaching

2.4.1. Creating to-scale models of the solar system

The following examples are intended to illustrate what modeling can mean for mathematics and physics lessons from an interdisciplinary perspective. The example of the proportional representation of the solar system emerges both in the interview material and in educational literature when discussing the topics of models and modeling. The challenge with such depictions lies in the fact that, in a visual representation (e.g., on a DIN-A4 sheet), the diameters and distances of the planets cannot be simultaneously depicted according to the proportions (in a linear representation). Therefore, various incorrect representations exist for this example, which might be suitable for initiating critical discussions in the classroom [18].

Solutions for accurate representations, initially in tangible form, may involve collecting suitable everyday objects (balls, berries, nuts, etc.) and placing them at the appropriate distances from each other. Another approach that emphasizes the modeling process is to use 3D printing technology to print spheres in a suitable size. Setting the maximum distance (Sun – Neptune) determines the conversion factor. With today's user-friendly software, such as Tinkercad [19], the modeling process becomes accessible to younger students at every step.

Another non-tangible but purely visual approach can involve the use of more advanced mathematics. By applying a logarithmic representation, it becomes possible to simultaneously depict distances and spacings on a sheet of paper. In combination with widely available software (PowerPoint or drawing software) and aesthetically pleasing visuals (such as those freely available from NASA [20]), attractive, creative, and instructive graphics can be generated (Fig. 2).

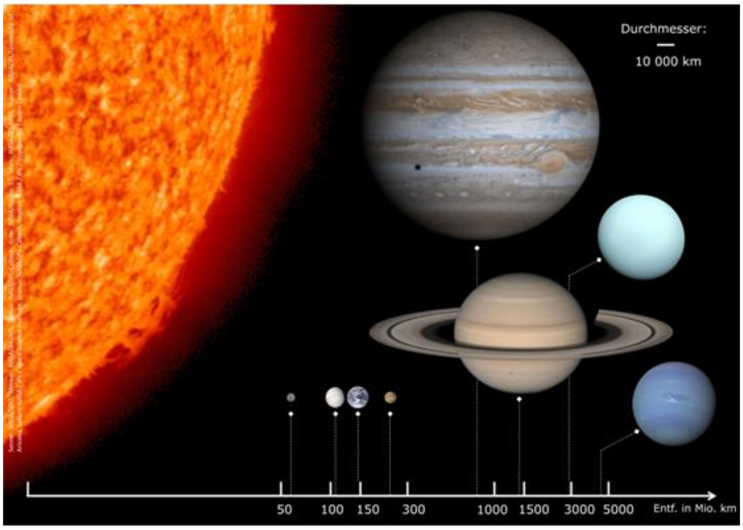

Figure 2 Representation of the solar system with correct diameters (linear scaling) and distances (logarithmic scaling)

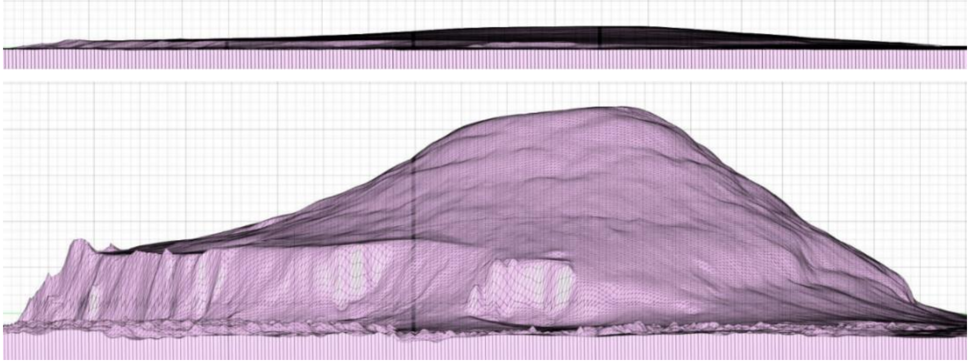

Figure 3 Exaggerated (below) and non-exaggerated (above) presentation of Olympus Mons as an example for the critical discussion of existing models (source: [20])

Examples from astronomy also allow for the critical discussion of widely established representations. For instance, there are often views of the surfaces of distant planets with an exaggeration in their representation, as commonly seen in geography. Figure 3 exemplifies two views of Olympus Mons, the tallest mountain in the solar system at 22 km in height. The lower sub-image is depicted with a 10-fold exaggeration (the strong exaggeration makes the distortion visible at some points, which was deliberately chosen in this example). Therefore, it does not provide an accurate picture of the true nature of the mountain. As a shield volcano, it has a diameter of 600 km, resulting in an average increase of only 5% at this maximum height of 22 km.

Apart from some steep slopes, the mountain would hardly be noticed as such during a hypothetical ascent.

Examples like these can clearly illustrate the purpose as well as the problems of such representations and sharpen the awareness of incorrect or distorted graphical depictions. This fosters a critical reflection on one's own or others' modeling processes.

2.4.2. Dynamic model building software

Another approach to modeling, which did not rely on the design of tangible objects, had its peak phase in Germany in the late 1990s. The approach was based on the use of dynamic modeling software. Schecker [21, p. 24] summarizes the core idea for the use of such software:

> *"Modeling is always also conceptualization through the application and consolidation of available knowledge or impulses for its modification".*

Or, as Bernshausen [22] puts it: "In contrast to ready-made, phenomenon-specific simulation programs, the user of modeling systems can and must perform the modeling – the physical mental work themselves."

The software's function is to represent the physical variables and their relationships with each other. The variables are connected by physical equations. The mathematical tools largely remain hidden since the software takes care of solving the equations, usually involving differential equations. This is not initially a disadvantage, as such differential equations are usually not part of the school curriculum. The use of modeling software is particularly suitable when the underlying systems are quite complex, and simple cause-and-effect relationships are no longer sufficient to describe the behavior. This is especially the case when feedback effects occur. As a simple example from mechanics, consider a vehicle rolling down a hill with a certain excess speed: The downhill force accelerates the vehicle further. At the same time, air friction increases with increasing speed, but also decreases with increasing air pressure (corresponding to the decreasing height when rolling down). Friction-related speed reduction, in turn, leads to a reduction in air friction and a decrease in deceleration [22]. The resulting speed in such a case, due to the mentioned feedback loops, can hardly be calculated using conventional means.

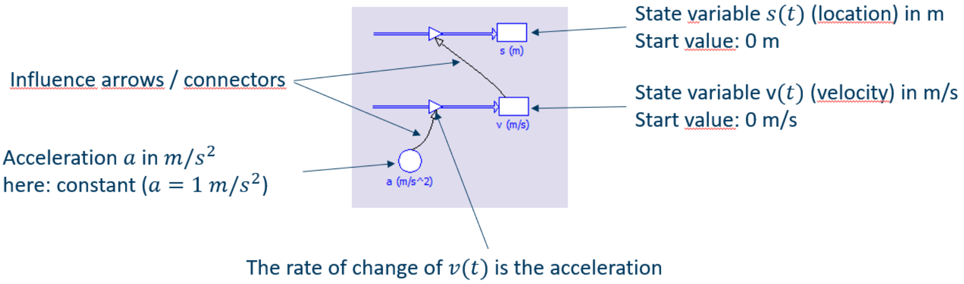

Figure 4 Illustration of the fall of a vehicle rolling downhill using the Coach modeling software (source [22], translation by the authors)

At such points, dynamic modeling software can provide a solution. Figure 4 exemplifies how a very simple model can be implemented in the "Coach"-software [23]. Starting from such a basic model, successive extensions can be added, such as an elevation profile approximating a real route. Figure 5 shows an extension of the model, illustrating the movement of the vehicle over different forces. Feedback and interactions between the current slope angle (German "Steigungswinkel" from the elevation profile), speed on air friction (German "F_Luft"), or speed on the motor force are apparent. Advanced models can recreate complex situations and answer questions such as the fuel consumption of a specific vehicle (with known engine power and drag coefficient). The results closely approximate real conditions, while complex traffic situations (frequent stops and starts) are not included in the model.

It should be noted that in this case, it is still primarily the elementary physical equations that are linked together. Therefore, it remains a modeling process that largely takes place on the conceptual level. The mathematical relationships within the equations and their interrelationships are relevant in this context.

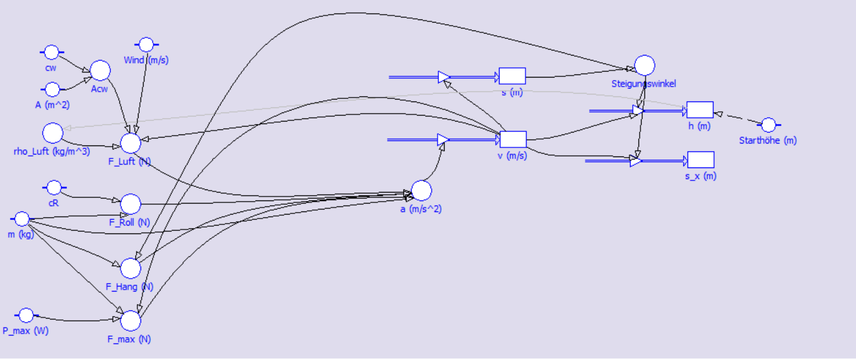

Figure 5 Illustration of the fuel consumption of a car ride with forces and an altitude profile (source [22])

3. Conclusions

Regarding the perspectives of mathematics and physics education on the concept of modeling, no fundamental differences were identified. In both subjects, modeling is seen as the transfer of real situations or nature into a model. While the product of modeling in physics education is often considered an object, in mathematics education, it predominantly appears in symbolic form. Both subjects share the characteristic that little attention is paid to the product of the modeling process, with this issue being much more pronounced in the context of mathematics education. Concerns about the ability of students to contribute creative ideas in the modeling process are evident in both subjects. Nevertheless, such creativity is deemed essential, as the process is understood as authentic problem-solving, aiming for novel solutions using open problem-solving approaches.

This observation presents a contradiction to relevant empirical findings where students self-assess their creativity (Westhoff and Heinicke, 2023). At the same time, the scientific-mathematical subjects, including mathematics and physics, are perceived as less creative in this study. The authors themselves point out the problem that the specific activities considered creative were inadequately captured. However, considering the generally positive attitude towards creativity in teaching, it seems desirable to enhance mathematics and physics education with elements perceived as creative.

For this to happen, it is necessary for teachers not to recognize the required creativity as an obstacle but rather accept its potential to enhance motivation. In this context, cooperation between mathematics and physics education could prove beneficial. By cooperation, does not necessarily refer to an interdisciplinary approach but rather a division of labor. In this sense, modeling in physics education could precede mathematical modeling. As discussed earlier, scholars like Schlichting do not necessarily view modeling as a quantitative process but emphasize conceptual considerations. Such modeling, for instance, through Black Box experiments, can go beyond the targeted creation of concrete models. Building on conceptual understanding, a qualitative approach can then be practiced, with mathematics education taking the leading role. Teachers believe that transferring real-life situations into mathematical models for non-physics scenarios is considered easier. Quantitative models of physical systems would conclude this sequence.

Due to the opportunity to incorporate creative elements into teaching, the tangible modeling in physics initially seems positive. However, considering that creativity and openness predominantly refer to aspects of tangible implementation and not to conceptual-physical or mathematical aspects, the value in terms of productive model construction appears questionable. Additionally, the framework conditions (i.e., the question of which degrees of freedom exist in the design) are largely unknown. The ultimate objective seems to be more focused on using the model as a product, as a tool to improve understanding.

On the organizational side, it is desirable to create specific opportunities in the curricula, involving open learning situations where students can develop their own solutions. In line with Schlichting, we consider it necessary to make the modeling process transparent for students and explicitly designate it as such. During the interviews, there was a case where a task, which could also be seen as a problem-solving task, was only recognized as an example of mathematical modeling after the interview concluded. Due to the interview questions, which did not explicitly introduce examples from the interviewers, it is unclear how many modeling tasks exist in the teaching that, while addressed in the classroom, may not be immediately recognized as having a modeling character by the teacher.

Regarding the requirements of the educational standards in both subjects, the handling of the model as a product of the modeling process is also relevant. The critical evaluation and further development of these models require, first and foremost, that these products be recognized as valuable outcomes and designated for further exploration. However, the statements of the teachers indicate that especially the mathematical models created in the process do not play a role in subsequent instruction. Especially at this point, the potential of integrating different subjects, such as Mathematics with Physics, becomes evident. Due to the different emphases on the process and the product, a balanced treatment can be achieved by combining both perspectives.

The examples for implementation in the classroom illustrate how modeling can be meaningfully combined with subsequent, productive handling of the model as a product across different school levels. Particular emphasis should be placed on the evaluation and critical discussion of the results of one's own and others' models in this context.

The material collected has proven to be a rich source of insights into the beliefs of teachers. So far, it has not been analyzed in terms of all possible aspects of the use of models and modeling. In particular, the evaluation of views on the characteristics of models and the type of models is still pending. In the course of the analysis, additional questions have arisen, but their answers could not be provided with the current methodological approach. Concrete questions for further investigations could include:

- How do beliefs about models and modeling influence teaching about models and modeling?
- How can modeling competencies be defined in the broad context of STEM education?
- What role does creativity play in the process of modeling? What influence does the perception of creativity by teachers and students have on the process of modeling?

A combination of a theoretical-conceptual approach with classroom observations and empirical assessments of students' perspectives will be necessary for this purpose.

Another open task is to identify additional examples, building on existing materials and tasks, through which open and creative modeling becomes possible in contexts that are meaningful for students. This is particularly important when it comes to the question of which specific tasks and problem solving approaches should be included in the STEM lessons to be implemented to ensure that the elementary concepts of model and modeling are taken into account in a subject-specific way.

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