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DEVELOPING A SYSTEM OF PRACTICAL EXERCISES TO ENHANCE MATHEMATICAL PROBLEM-SOLVING COMPETENCIES FOR FIFTH-GRADE STUDENTS THROUGH REALISTIC MATHEMATICS EDUCATION

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Abstract. This paper develops a system of practical exercises to enhance the mathematical problem-solving competencies of fifth-grade students using Realistic Mathematics Education. The novelty of this study lies in creating tasks grounded in meaningful contexts, aimed at fostering mathematization and deeper conceptual understanding. Unlike traditional exercises, these tasks engage students in applying mathematics to situations relevant to their experiences. Additionally, the paper provides a framework for designing similar tasks, contributing valuable resources for elementary mathematics education under the RME framework.
Keywords: a system of practical exercises, problem-solving competencies, RME.

1. Introduction

Realistic Mathematics Education (RME) is one of the mathematics education trends being focused on globally and in Vietnam. A key feature of RME is its emphasis on teaching through context, enabling students to apply mathematics, and making practical situations integral to the learning process [1]. These situations serve as a foundation for developing concepts, tools, and mathematical knowledge and as a context for students to apply the mathematics they have learned. Two core principles of RME are: "Mathematics is closely connected with everyday life" and "Mathematics is a result of human activity" [2]. Therefore, learning mathematics is not just about listening, understanding, and passively receiving knowledge from the teacher. Rather, it involves transforming practical problems into solutions and constructing concepts, processes, and algorithms, all under the guidance and facilitation of the teacher. When students observe the relationship between mathematics and real life, it increases their motivation, confidence, and love for the subject.

After many years of development, RME has become a widely adopted approach to mathematics education in the Netherlands. The Freudenthal Institute at Utrecht University, along with other research institutions in the Netherlands, has made significant contributions to the advancement of RME. "Approximately 75% of Dutch primary schools currently use textbooks based on the philosophy of RME" [3]. The Hewet project (1981-1985) developed a mathematics curriculum for secondary schools and for students in higher education, particularly in the social sciences and humanities. The effectiveness evaluation strategy of RME was analyzed and further developed in Van den Heuvel's 1996 doctoral dissertation. This pedagogical approach has also been incorporated into many secondary school mathematics textbooks in the United States under the name "Mathematics in Context" (abbreviated as MiC), one of the textbook series that links mathematics with practical situations. Consequently, RME has been studied and applied in various countries, serving as both a curriculum development theory and a teaching theory. The widespread adoption of RME has been widely recognized, and as of a 2016 publication, at least 15 countries, including the United Kingdom, the United States, Singapore, and Indonesia, have conducted research and actively implemented this theory in their mathematics education systems. This underscores the importance and global impact of RME on the advancement of education worldwide [3].

Although the majority of research on RME originates from the Netherlands, there is substantial evidence of its influence in various other countries. International studies on RME publications highlight its focus on enhancing mathematical competencies and fostering critical thinking skills. The research conducted in Greece has shown that RME can support the development of mathematical abilities in children aged 4 to 6 [4]. Furthermore, several studies have examined the implementation of RME in the context of mathematics education in Vietnam. Notable contributions include works such as Exploring Realistic Mathematics Education connected to practical applications and applying it to develop practical exercises in teaching Mathematics [5], Applying RME in Teaching: Challenges, Principles, and Recommendations [6], Approaching and proposing some measures for applying RME theory in teaching and learning mathematics in Vietnam [7], and Some fundamental methodological issues relating to teaching the multiplication of two natural numbers to primary students [8], etc. These studies collectively aim to clarify the methods for implementing RME in mathematics instruction and offer recommendations for its potential integration into mathematics education in Vietnam.

Problem-solving competencies are one of the essential competencies that should be fostered in students, and for a long time, problem-solving competencies have been at the core of mathematics teaching and learning [9]. However, students' mathematical problem-solving competencies remain limited and need improvement [10], [11]. Therefore, proposing strategies to promote students' problem-solving competencies is necessary and aligns with the objectives of the General Education Mathematics Curriculum [12].

To develop this competence for students, several studies have designed exercise systems within the mathematics curriculum, such as Improving the Students' Mathematical Problem-Solving Ability by Applying Problem-Based Learning Model in VII Grade at SMPN 1 Banda Aceh Indonesia [10], The problem-solving competence on linear equations of 10-grade students [11]. Moreover, some researchers have applied RME to enhance students' mathematical problem-solving competencies, including: Designing exercises containing real-life situations in teaching mathematics in high schools [13], Improving conceptual understanding and problem-solving in mathematics through a contextual learning strategy [14], The Effect of Metacognitive-Based Contextual Learning Model on Fifth-Grade Students' Problem-Solving and Mathematical Communication Skills [15], Designing Practical Situations with Real Contexts in Teaching Mathematics in Primary Schools [16]. These studies leverage and utilize a system of exercises to develop problem-solving competence based on the RME approach for students at various educational levels.

Primary education is the first formal stage in Vietnam's general education system and plays an important role in laying the foundation for students' personality development. In mathematics, particularly in the fifth grade, innovations in competency-based learning methods have also been introduced.

2. Content

2.1. Theory of realistic mathematics education

The theory of RME emerged in the 1960s in the Netherlands as a response to the country's mathematics education, which was dominated by mechanistic teaching methods and an approach that viewed mathematics as a purely scientific discipline. The program lacked practical connections to everyday contexts, preventing students from understanding the applied significance of mathematics and thus reducing their interest in learning the subject. In this context, several mathematicians and mathematics educators called for a reform in mathematics education, leading to the birth of the RME school of thought. H. Freudenthal was a strong advocate for innovation in mathematics education. He and his colleagues published the first works outlining the foundational ideas of RME [3].

According to Freudenthal's interpretation of RME, mathematics is a human activity, and it should not be learned as a closed system, but rather as an activity of mathematizing practical situations, and where possible, even mathematizing mathematics itself. Students should be encouraged to "create opportunities to reinvent mathematics by mathematizing concrete situations or mathematical relationships and processes". The materials used by students must be realistic and relevant to them, ensuring that they can apply mathematics to practical situations they encounter, and see the meaning and connections of mathematics in their daily lives. The goal of RME is to make "school mathematics more practical, relevant, and meaningful" for a broader range of students [3].

Realistic Mathematics Education is a domain-specific instruction theory in mathematics teaching that ensures rich, everyday teaching situations are prominently featured throughout the learning process. These situations serve as "a source for initiating the development of mathematical concepts, tools, and procedures and as a context in which, at later stages, students can apply their mathematical knowledge, gradually making it more formal and general, with less reliance on specific contexts" [17]. In RME, practice serves as a resource for the teaching and learning of mathematics. While RME situations are understood as "everyday" situations, which are problematic situations that students can imagine, they are not limited solely to external applications, practical contexts still play a critical role in RME [3].

According to RME, students need to "learn mathematics in context", and thus teaching according to RME requires changes in instructional methods and learning organization. Rather than merely transmitting knowledge, RME encourages designing learning environments that challenge students, promote their independence, stimulate deeper thinking, and thereby improve their problem-solving competencies.

2.2. Mathematical problem-solving competency

According to the authors in ref. [12], competency is defined as an individual attribute formed and developed through innate qualities and the processes of learning and training, which allows a person to mobilize and integrate knowledge, skills, and other personal attributes such as interests, beliefs, willpower, etc., to successfully perform a specific activity and achieve desired results under given conditions.

PISA has broken down the problem-solving process into six stages: Understanding the problem; Describing the problem; Representing the problem; Solving the problem; Reflecting on the solution method; and Communicating the solution method [18].

A problem solver may partially identify the action goal, but not immediately know how to achieve it. Comprehension of the problem situation and gradual reasoning toward that goal based on planning and logical inference constitute the problem-solving process. According to the authors in ref. [19], problem-solving refers to the process in which an individual utilizes their existing knowledge, skills, and understanding to meet the demands of unfamiliar situations encountered. To support students in finding problem-solving approaches, the authors in ref. [20] proposed ten strategies that learners can use in mathematical problem-solving: working backward, searching for a pattern, approaching the problem from a new perspective, solving a simpler but similar problem, considering special cases, illustrating with diagrams, guessing and checking, considering all possible outcomes, organizing data, and logical reasoning.

Problem-solving competency is the ability of an individual to effectively use cognitive processes, actions, attitudes, motivations, and emotions to resolve problematic situations where standard procedures or solutions are not readily available [21]. Thus, students' problem-solving competency can be understood as the ability to coordinate and apply their personal experiences, knowledge, and skills from different subjects in the curriculum to successfully solve problematic situations in learning and everyday contexts with a positive attitude.

From the concept of competency and problem-solving competency, we argue that mathematical problem-solving competency refers to the process in which students use their knowledge, skills, and experiences to solve mathematical situations in both learning and practical contexts. According to the authors in ref. [12], mathematical problem-solving competency consists of the following components:

(1) Identifying and recognizing problems that need to be addressed through mathematics;

(2) Selecting and proposing problem-solving methods or solutions;

(3) Applying relevant mathematical knowledge and skills (including tools and algorithms) to address the given problem;

(4) Evaluating the proposed solution and generalizing it to similar problems.

2.3. Developing a system of practical exercises to enhance mathematical problem-solving competencies for fifth-grade students through the RME

2.3.1. Practical exercises

According to the authors in ref. [21], a practical problem pertains to human activities, primarily productive labor, aimed at creating the necessary conditions for societal existence. The Oxford Advanced Learner's Dictionary defines a practical problem as one that deals with things that exist or are happening; these are not imaginary scenarios [22]. According to ref. [23], "A practical problem is a problem whose need for resolution arises directly from practical human contexts". Furthermore, Hang, Quyet, and Thanh identify two types of practical problems:

(i) Problems connected to practical situations: These problems involve everyday contexts within either their premises or conclusions or in other words, they are problems with a real, situational context.

(ii) Quasi-practical problems (also known as semi-realistic problems): These problems are based on hypothetical situations that could potentially occur in human activities, where certain elements of the premises or conclusions are hypothetical.

Based on these perspectives, we define practical problems as those related to human activities, particularly in labor and production, where these activities are significant and require solutions.

In the theory of RME, two closely related terms are:

- Context problem [24]: These are problems that are authentic to learners' experiences. While traditionally, context problems were mainly understood as having practical applications, used primarily in the final stages of teaching for reinforcement, in RME theory, context problems can be used throughout the entire learning process. Originally, RME focused on elementary school mathematics, but Gravmeijer and Doorman extended its application to higher levels of education, such as calculus in secondary education.

- Task context [25]: The most important feature of a task context is that it must be suitable for mathematization, where learners can imagine and grasp the situation, allowing them to apply their knowledge and personal experiences. An ideal task context should:

+ Be meaningful to learners, i.e., it should be interesting, engaging, accessible, and pose a challenge that naturally generates the need for a solution.

+ Be informative, providing teachers with insights into learners' knowledge, experiences, strategies, and skills.

+ Be open-ended, allowing learners to personalize their solutions, reflecting their mathematization process. It should emphasize the learner's strengths, focusing on what they have mastered rather than testing what they do not yet know or are limited in.

Thus, whether the problem is practical or quasi-practical, it must be based on relevant, situational information and data. The context in RME theory accepts and even encourages, purely mathematical data as long as it is relatable to learners' knowledge and experience. We believe that practical exercises are those that are formulated using data or contexts familiar to learners' existing knowledge and experiences, allowing them to utilize their available resources for mathematical activities at various levels. Practical exercises can be used in various teaching processes, from motivation-driven tasks to guiding homework assignments.

2.3.2. The role of a system of practical exercises in developing mathematical problem-solving competencies through the RME

Practical problems play a crucial role in developing students' mathematical problem-solving competency through the following aspects:

(i) Problem-Solving Context: Practical problems present situations that provoke problem-solving, requiring students to mobilize their experiences, background knowledge, and mathematical understanding to develop solutions. This process not only deepens students' mathematical knowledge but also enhances their understanding of various practical contexts.

(ii) Engaging Mathematical Knowledge: Solving practical problems makes mathematical knowledge more dynamic and engaging, stimulating students' interest, initiative, and creativity, and fostering a passion for learning mathematics.

(iii) Connecting Theory and Practice: Solving practical problems bridges theoretical knowledge and practical application, helping learners better understand how mathematics is applied in various contexts, thereby reinforcing and retaining the learned concepts.

Designing practical exercises for mathematics requires attention to several factors to ensure their effectiveness in developing mathematical problem-solving skills within the framework of Realistic Mathematics Education for elementary students. To achieve this, the following aspects must be considered:

- Ensure alignment with curriculum objectives, knowledge, and skills standards, as well as the goal of fostering students' ability to apply knowledge in practical situations.

- Ensure the accuracy, scientific validity, and modern relevance of mathematical content and related scientific disciplines, providing sufficient data for problem-solving.

- Design exercises that are closely related to students' daily experiences and learning backgrounds.

- Ensure a systematic and pedagogically logical structure in the exercises.

2.3.3. Some types of practical problems and steps to design practical exercises

According to ref. [16], based on the cognitive characteristics, psychological development, and life experiences of elementary students, there are four common types of practical situations that teachers and students can utilize when designing contextual scenarios for teaching elementary mathematics:

(i) Situations with a personal context: These relate to activities involving the students themselves.

(ii) Situations with a family-related context: These reflect activities within the family, appropriate for students' age.

(iii) Situations with a school context: These involve activities taking place in the school environment where students are studying.

(iv) Situations with a community context: These reflect activities happening within the students' broader community.

Additionally, 16] outlines four steps for designing practical exercises with contextual relevance:

- Step 1: Define the content and objectives of the lesson.

- Step 2: Identify relevant contextual scenarios connected to the mathematical content or which implicitly express this knowledge.

- Step 3: Select an appropriate, practical context, collect necessary data, and pose the "practical situation with contextual relevance".

- Step 4: Adjust the situation if necessary.

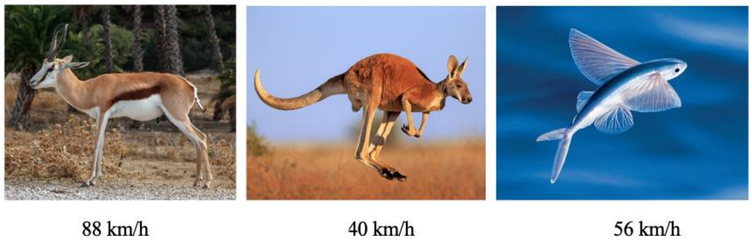
2.3.4. Illustrative example

Mathematical problems at the primary level can be categorized into the following specific groups:

- Group 1: Problems with numbers and operations with natural numbers.
- Group 2: Problems with fractions, ratios, and decimals.
- Group 3: Problems with uniform motion.
- Group 4: Problems on geometry.
- Group 5: Problems with applying quantities.

Below are some exercise systems according to groups of exercises that we have collected from many different sources and built some problems:

Exercise 1: Minh and Nam are watching a TV show called *Exploring the World* with the theme of *The Animal Kingdom*. When they hear the host say, "A Springbok antelope can run at a maximum speed of 88 km/h. A Kangaroo can run at a speed of 40 km/h. A flying fish can move at a speed of 56 km/h...," Minh turns to Nam with a question: "Can you guess which is faster over one hour, the Springbok or the Kangaroo, and by how many kilometers?" Nam then challenges Minh: "What is the speed of the flying fish in meters per second?" Can you help them solve each other's questions?



Firgue 1. (Source: Internet)

Step 1: Define the content and objective of the lesson.

The objective of this lesson is to convert speed units from kilometers per hour (km/h) to meters per second (m/s) and vice versa.

Step 2: Identify relevant contexts linked to mathematical concepts or those implicitly containing them.

When incorporating practical contexts into lessons on uniform motion for elementary students, teachers should relate the numerical values to practical situations involving speed, distance, time, etc., of objects in everyday life. Teachers must select appropriate practical exercises, ensuring that the situations presented are realistic and relatable to students' experiences.

Step 3: Select a suitable context, gather data, and pose the "practical situation with contextual relevance".

Choose an accessible context for students. Present the scenario: Minh and Nam are watching *Exploring the World* with the theme of *The Animal Kingdom* on TV. When they hear the host say, "A Springbok antelope can run at a maximum speed of 27 m/s. A Kangaroo can run at a speed of 40 km/h. A flying fish can move at a speed of 56 km/h...," Minh challenges Nam: "Which one is faster over one hour, the Springbok or the Kangaroo, and by how many kilometers?" Nam then challenges Minh: "What is the speed of the flying fish in meters per second?" This situation falls into group 4 mentioned in section 2.3.2.

Step 4: Adjust the situation (if necessary).

Conduct a trial where students solve the situation, gather feedback from colleagues and experts, and make adjustments to the scenario if needed.

Exercise 2: Quan's family drives from Nam Dinh city to Hai Phong city departing at 9:25 AM and arriving at 11:45 AM, traveling at a speed of 70 km/h. Along the way, the car stops for a 20-minute break. How long is the distance between Nam Dinh city and Hai Phong city in kilometers?

Figure 2. (Source: Internet)

Step 1: Define the content and objective of the lesson

The objective of this lesson is to calculate the speed of uniform motion to solve various practical problems.

Step 2: Identify relevant contexts linked to mathematical concepts or those implicitly containing them

When incorporating practical contexts into lessons on uniform motion for elementary students, teachers should relate numerical values to practical situations involving speed, distance, time, etc., of objects from daily life. Teachers must select appropriate practical exercises, ensuring that the situations presented are realistic and familiar to the students.

Step 3: Select a suitable context, gather data, and pose a "practical situation with contextual relevance"

Choose a situation that is easily accessible to students. Present the following exercise: Quan's family drives from Nam Dinh to Hai Phong, departing at 9:25 AM and arriving at 11:45 AM, traveling at a speed of 70 km/h. Along the way, the car stops for a 20-minute break. How long is the distance between Nam Dinh city and Hai Phong city in kilometers? This practical situation corresponds to group 2 as mentioned in section 2.3.2.

Step 4: Adjust the situation (if necessary)

Test the situation by organizing students to solve it, gather feedback from colleagues and experts, and make adjustments to the scenario if necessary.

Exercise 3: During the Physical Education test, the teacher assigned the class monitor to record the distance and time of students in group 3 who registered for the running assessment. The class monitor collected the following data:

No.	Full Name	Time	Distance
1	Do Minh Nhat	37 seconds	100 m
2	Tran Hai Nam	32 seconds	100 m
3	Nguyen Hai Đang	35 seconds	100 m
4	Mai Thanh Ha	33 seconds	100 m
5	Ho Thanh Chi	39 seconds	100 m

Step 1: Define the content and objective of the lesson

The objective of this lesson is to calculate the speed of uniform motion to solve various practical problems.

Step 2: Identify relevant contexts linked to mathematical concepts or those implicitly containing them

When incorporating practical contexts into lessons on uniform motion for elementary students, teachers should relate numerical values to practical situations involving speed, distance, time, etc., of objects from daily life. Teachers must select appropriate practical exercises, ensuring that the situations presented are realistic and familiar to the students.

Step 3: Select a suitable context, gather data, and pose a "practical situation with contextual relevance"

Choose a situation that students can easily relate to. Present the following exercise: During a Physical Education test, the teacher assigns the class monitor to record the distances and times of students from group 3 who registered for the running test. Calculate the speed of each student. This practical situation corresponds to group 3 as mentioned in section 2.3.2.

Step 4: Adjust the situation (if necessary)

Test the situation by having students solve it, gather feedback from colleagues and experts, and make adjustments as needed.

Below is a system of exercises grouped by category:

Group 1: Problems with numbers and operations with natural numbers

Problem 1: Nam's average score for the first three math tests is 5 points. How many points does Nam need to score on the next test so that his average score over all four tests will be 6 points?

Problem 2: Minh has 100,000 VND to buy pens. There are two types of pens: Type I costs 16 000 VND each, and Type II costs 8 000 VND each. Determine the maximum number of pens Minh can buy if:

- She only buys Type I pens.

- She only buys Type II pens.
- She buys an equal quantity of both types of pens.

Problem 3: To prepare for the new school year, Thu's mother gave her 200,000 VND to buy pens and notebooks. Each pen costs 16 000 VND, and each notebook costs 32 000 VND. Find out:

- The maximum number of pens Thu can buy if she only buys pens.
- The maximum number of notebooks Thu can buy if she only buys notebooks.
- The maximum number of pens and notebooks Thu can buy, with equal quantities of both.

Problem 4: A train needs to transport 1 800 passengers. Each carriage has 12 compartments, and each compartment has 6 seats. What is the minimum number of carriages required to transport all passengers?

Problem 5: To transport 229 tons of goods, two types of trucks are used. Type I trucks can carry up to 5 tons, and Type II trucks can carry up to 8 tons. What is the minimum number of trucks required if:

- Only Type I trucks are used.
- Only Type II trucks are used.

Group 2: Problems with fractions, ratios, and decimals

Problem 1: The first tractor takes 9 hours to plow a field, and the second tractor takes 15 hours to plow the same field. After the first tractor works for 6 hours, it stops, and the second tractor takes over to complete the plowing. How long did the second tractor work? *(Source: Internet)*

Problem 2: An's sister, after graduating from university and receiving her first paycheck, decides to spend 3/5 of her salary on family expenses and 1/3 of her salary on gifts for her parents. The remainder she puts into savings. What fraction of her salary is saved?

Problem 3: A teddy bear costs 450,000 VND in a shop. For Christmas Day, the shop offers a 20% discount on all items. What is the price of the teddy bear after the discount?

Problem 4: Due to the COVID-19 pandemic in 2020, many factories faced difficulties. A garment factory had a balance of 2.3 billion VND in February 2020. By March 2020, after switching to mask production for export, its balance increased to 3.5 billion VND. How much did the factory's balance increase in March 2020? *(Source: Internet)*

Problem 5: The ticket price for children to watch a movie is 65,000 VND per ticket. After reducing the ticket price, the number of attendees increased by 25%, and the total revenue increased by 12.5%. What was the ticket price after the discount?

Group 3: Problems with uniform motion

Problem 1: A tourist travels from Hanoi to visit Tran Temple in Nam Dinh city. In the first stage, they cover 1/2 of the distance from Hanoi to Nam Dinh, in the second stage, they cover 1/6 of the distance, and in the third stage, they cover 1/5 of the distance. They are then left with 12 km to reach Nam Dinh. Determine the total distance the tourist must travel from Hanoi to Nam Dinh.

Problem 2: A border guard is on a patrol mission from station A to checkpoint B and then back to station A. The distance from station A to checkpoint B is 9 km, which consists of three segments: the first segment is downhill, the second is a flat road, and the third is uphill. The guard took a total of 3 hours and 21 minutes to travel from station A to checkpoint B and then return. It is known that his speed is 4 km/h uphill, 6 km/h downhill, and 5 km/h on the flat road. How long is the flat segment of the road?

Problem 3: To prepare for a visit to Thay Pagoda, Nam and An made a plan to travel from their home in Hanoi. Nam started his trip from Hanoi at 8:00 a.m. at a speed of 10 km/h. An wants to arrive at Thay Pagoda 15 minutes after Nam. What time should An start from Hanoi, knowing that his speed is 15 km/h and the distance from Hanoi to Thay Pagoda is 30 km?

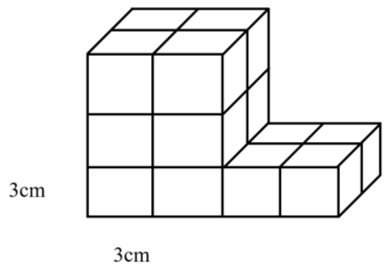
Problem 4: The distance between Hanoi and Hai Duong is 58 km. At 8:00 AM, a cyclist departs from Hai Duong for Hai Phong at a speed of 20 km/h. At the same time, a car departs from Hanoi for Hai Phong via Hai Duong at a speed of 50 km/h. How long will it take the car to catch up to the cyclist, and how far from Hanoi will that occur?

Problem 5: Every day, Mr. Khanh bicycles to work at a speed of 15 km/h. This morning, due to a matter, he started 10 minutes late. To arrive at work on time, he calculated that he would need to increase his speed to 18 km/h. Determine the distance from Mr. Khanh's house to his workplace.

Group 4: Problems with Geometry

Problem 1: Every weekend morning, Bao runs 0.314 km on a circular path around Vi Xuyen Lake, which has a circular shape with a diameter of 10 m. How many laps does Bao complete each morning?

Problem 2: Dang's toy set contains 15 identical cubes, each with an edge length of 3 cm.



Firgue 3. (Source: Internet)

(i) Can Dang arrange these cubes into a rectangular prism? If so, how many different ways can he do it, and what are the possible dimensions of the prism?

(ii) Calculate the volume of the rectangular prisms formed from the 15 cubes.

Problem 3: A swimming pool has a length of 18 m, a width of 7 m, and a depth of 2.75 m. How many tiles are needed to tile the bottom and sides of the pool, assuming each tile is 25 cm long and 20 cm wide, and the thickness of the grout is negligible?

Problem 4: Every afternoon, Minh and Son run 20 laps around a semi-circular garden with a diameter of 40 m. Determine the total distance they run.

Problem 5: A rectangular house floor has a semi-perimeter of 98 m, with a width of 8 m. The floor is to be tiled using square tiles with a side length of 3 dm. Find the number of tiles needed to cover the floor, assuming the space between tiles is negligible.

Group 5: Problems with applying quantities *(Source: Internet)*

Problem 1: Body Mass Index (BMI) is a measure commonly used to assess whether a person’s body is within a healthy range, underweight, or overweight, based on their height and weight. The BMI is calculated using the formula $BMI = \frac{m}{h^2}$, where m is the body mass in kilograms and h is the height in meters. For sixth-grade students in Asia, the BMI categories are as follows:

- BMI < 15: Underweight
- 18 ≤ BMI < 23: Normal
- 23 ≤ BMI < 30: Overweight
- 30 ≤ BMI < 40: Obesity (moderate)
- BMI ≥ 40: Obesity (severe)

A fifth-grade student is 140 cm tall and weighs 40 kg. What is this student's BMI category?

Problem 2: A wall clock has the following feature: When the minute hand points to 12, the clock strikes the number of times corresponding to the hour hand. How many times does the clock strike in one day?

Problem 3: Hong plans to make 100 kg of jam using strawberries and sugar. The recipe requires 2 kg of strawberries for every 3 kg of sugar. How many kilograms of strawberries and sugar does Hong need to make the jam?

Problem 4: A book contains 315 pages. How many digits are needed to number the pages, starting from page 1?

Problem 5: A ripe banana weighing 120 grams contains 0.3% fat and 0.42% potassium. Calculate the total weight of fat and potassium in the banana.

2.3.5. Guidelines for using a system of practical exercises to develop mathematical problem-solving competency through the RME

To effectively use the exercises system aimed at developing problem-solving competencies based on RME with the 25 specific exercises provided, ensuring appropriate and effective application in the teaching process for fifth-grade students, teachers should note the following points:

- Identify the criteria for selecting and using additional exercises to develop problem-solving competencies based on RME.

+ Based on the objectives and content of the specific mathematics lesson being taught, particularly exercises that can enhance problem-solving abilities through the lens of RME, teachers should select appropriate exercises.

+ Relying on general mathematical knowledge and the learning abilities of students (to group students with suitable exercises), as well as the specific teaching conditions of their class (classroom space, projectors, computers, etc.).

- Direction for selecting and using additional exercises to develop problem-solving competencies based on RME:

+ Teachers should choose from the pool of exercises and specific problems that best align with the aforementioned criteria to supplement or replace the exercises selected from the textbook (if the exercises in the textbook are not entirely suitable). The number of selected exercises should depend on the lesson's time frame, typically ranging from 1 to 3 exercises, depending on the student group.

+ For each additional exercise students are required to complete, teachers must clarify the purpose and tangible benefits of doing the exercises to direct students' learning and promote the development of problem-solving competencies through RME.

- Organizational methods for students to complete exercises that foster problem-solving competencies through RME:

+ Teachers can either write the exercises on the board or copy them onto worksheets for students to solve individually or in groups. If students are grouped, teachers should divide them according to their proficiency levels (average, fairly good, or excellent) to select appropriate exercises of varying difficulty, thus increasing the number of exercises used.

+ Based on the requirements and specific content of the exercises, teachers may organize problem-solving activities through games or competitions to increase students' engagement and enhance their active learning participation.

To enhance problem-solving competency through a system of practical exercises, it's crucial to develop a comprehensive and diverse exercise bank that progresses from foundational to complex problems. This system should incorporate varied problem types, including both closed-ended and open-ended questions while emphasizing specific problem-solving strategies. Encourage metacognition by prompting students to reflect on their problem-solving processes, and integrate collaborative exercises to foster teamwork and communication skills. Implement immediate feedback mechanisms, timed practice sessions, and cross-disciplinary applications to broaden understanding and improve efficiency. Regular assessment and adaptation of the exercise system, along with the encouragement of creative problem-solving and the utilization of technology, will ensure continuous improvement in analytical reasoning, critical thinking, and innovative solution development. This systematic approach not only enhances mathematical proficiency but also cultivates valuable skills applicable across various disciplines and practical situations.

3. Conclusions

The development of mathematical problem-solving competencies is one of the key instructional orientations being emphasized in the current educational context. This paper proposes the construction of practical exercises aimed at enhancing MPSC through the approach of RME and illustrates this process with practical tasks designed for fifth-grade students. In teaching practical problems, teachers should flexibly apply teaching methods and procedures to help students approach practical issues in various ways, identify the problems to be solved, and determine the appropriate solutions. This will lead to a deeper understanding of the applications of mathematics in practical contexts.

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