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DEVELOPING MATHEMATICAL LITERACY THROUGH AN INSTRUCTIONAL PROCESS: A GRADE 9 CASE STUDY ON SOLVING SYSTEMS OF LINEAR EQUATIONS IN TWO VARIABLES

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Abstract. This research develops and illustrates an instructional process integrating Realistic Mathematics Education (RME) principles with the DAPIC problem-solving cycle (Define, Assess, Plan, Implement, Communicate) to enhance mathematical literacy among Grade 9 students. Addressing gaps in existing frameworks, the model, grounded in the PISA framework, emphasizes mathematical reasoning across all phases while embedding critical 21st-century skills. The process begins with the realistic problems to engage students, progresses through collaborative solution exploration and discussion of diverse strategies, and culminates in formalizing mathematics and applying knowledge to new contexts. A Vietnamese classroom case study on solving systems of linear equations in two variables demonstrates how students transition from informal methods to structured algebraic techniques. Guided by targeted teacher prompts and collaborative discussions, students deepen both conceptual understanding and procedural fluency. The findings suggest that this integrated RME–DAPIC approach not only aligns with international literacy goals but also effectively cultivates the core dimensions of mathematical literacy within realistic contexts and supports students' development as reflective and adept problem-solvers in secondary mathematics education.

Keywords: mathematical literacy, instructional process, RME, DAPIC, systems of linear equations.

1. Introduction

"We live in an era of universal mathematical education" [1]. According to UNESCO (1990), fundamental learning tools and the basic learning content are educational opportunities that every individual should be able to benefit from [2]. In order to meet the basic demands of everyday living, researchers assert that mathematical literacy (ML) is essential for all individuals [3]. Specifically, McCrone & Dossey (2007) contend that ML prioritizes "bringing relevance and deeper understanding to mathematical learning situations that empower individuals relative to their present and envisioned needs" [4, p. 32]. The focus of ML applies to everyone, not just those pursuing careers in science or engineering [4]. Therefore, developing ML for all students is a critical mission.

ML enables individuals to recognize how mathematics influences all areas of life, and reflects the ability to formulate, employ, and interpret and evaluate mathematics across diverse contexts [5]. Context distinguishes mathematics from ML, as mathematics focuses on technical concepts and procedures, whereas ML is grounded in real-world contexts that emphasize practical application [6]. Instead of assessing mastery of specific mathematical topics, the Program for International Student Assessment (PISA) emphasizes contextual questions that reveal how well students can apply their knowledge and demonstrate their mathematical reasoning [7]. This reinforces the value of using real-life contextual problems to cultivate ML. By incorporating realistic contexts, Realistic Mathematics Education (RME) effectively exposes students to problems that enhance their ML [8]. From the inception of RME, Freudenthal (1968) stressed that when children learn mathematics in isolation from their experiences, they are likely to forget it quickly and struggle with its application [9]. In this sense, RME serves as an effective strategy for implementing a "problem-oriented curriculum" [10, p. 6] by engaging students with non-routine problems that demand "some degree of independence, judgment, originality and creativity" [11, p. xi].

Although most teachers acknowledge the importance of non-routine, real-world problems for developing students' ML, they find learners are often confused by unfamiliar question types; consequently, they rely on traditional methods and hesitate to adopt a literacy-focused approach because it demands substantial changes to their established practices [12]. Traditional methods, which rely on rote memorization of rules and formulas unrelated to real-world experiences, fail to foster ML, making the search for a more effective instructional process essential [13].

Various instructional processes have been proposed to enhance students' ML. Sumirattana et al. (2017) integrated RME with the DAPIC problem-solving cycle (define, assess, plan, implement, and communicate) to develop a five-step instructional process for improving ML among 9th-grade students [13]. Çakıroğlu et al. (2024) subsequently employed a five-step framework to guide their study on integrating Virtual Reality into RME to develop ML skills [14]. Although existing instructional processes offer a broad framework for cultivating students' ML, they often only partially address its dimensions. For example, the model proposed by Sumirattana et al. (2017) conceptualizes ML in terms of only two components: knowledge and competency [13]. Meanwhile, Çakıroğlu et al. (2024) draw on earlier OECD-PISA frameworks comprising three dimensions: formulate, employ, and interpret and evaluate [14]. To improve the model, the present study adopts the latest PISA framework, which expands the definition of ML by incorporating mathematical reasoning and key 21st-century skills required for students to become "constructive, engaged and reflective 21st Century citizens" [15, p. 22], as will be further discussed in the subsequent section.

The "Change and Relationships" domain is one of PISA's four mathematical content areas used to assess ML. Both natural and human-designed systems exhibit numerous temporary and enduring interconnections where components influence one another and evolve; consequently, equations are central tools for describing, modelling, and interpreting these dynamic phenomena [15]. Learning equations, particularly linear equations, can be difficult for many students due to their abstract nature [16]. Bosque (2025) found that students frequently struggle to select appropriate methods (substitution, elimination, or graphing), often confusing procedural steps when solving systems of linear equations, and often perceive these tasks as tedious and disconnected from real-world contexts [17]. Previous studies have addressed the teaching of systems of linear equations by utilizing realistic situations to transition students from informal methods to formal algebraic tools [18], and by using real-world problems to develop specific competencies such as mathematical modeling [19] as well as mathematical thinking and reasoning [20]. Building on these perspectives, our study affirms the foundational role of realistic contexts and the necessity of cultivating specific cognitive competencies when students solve linear systems. To provide a more structured pedagogical pathway, we refine these approaches by explicitly integrating RME principles with the DAPIC problem-solving cycle. This integration creates a comprehensive five-step instructional process that simultaneously addresses all dimensions of ML and 21st-century skills, illustrated through a detailed case study on systems of linear equations, which is a topic that offers rich opportunities for contextualized problem solving in the 9th-grade mathematics curriculum in Vietnam.

2. Content

2.1. Mathematical literacy and relevant theories

2.1.1. Mathematical literacy

Educators and researchers lack a consensus on the precise meaning of ML [3]. This concept involves widely debated definitions, often linked to the ability to address the quantitative aspects of life, and has been referred to by various terms such as quantitative literacy, numeracy, ML, quantitative reasoning, or simply mathematics [21]. Some researchers, including Goos et al. (2012), note that these terms are often used interchangeably [22], although Niss & Jablonka (2020) argue that ML is more closely associated with schooling contexts, whereas numeracy pertains to applying mathematical skills in adult life contexts [23]. This supports the perspective of Cockcroft's (1982) perspective, which asserts that numeracy is primarily utilized in adult mathematics education programs and generally denotes an individual's ability to manage the practical mathematical demands of everyday life [24]. Despite these nuances, the contested nature of these concepts remains evident; as Coben et al. (2003) point out, numeracy is "a deeply contested" (p. 7) and "notoriously slippery concept" (p. 9), with its definition continuing to vary [25]. The choice of terminology often depends on regional preferences. For instance, "numeracy" is normally used in English-speaking countries such as the UK, Canada, South Africa, Australia, and New Zealand, while in other international contexts, it is referred to as ML or quantitative literacy [26].

Notably, OECD's (2023) definition of ML is described as an individual's capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts. It includes concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to know the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged, and reflective 21st Century citizens [15, p. 22], which closely aligns with the International Life Skills Survey's (ILSS) perspective [27]. A comprehensive framework is proposed by the OECD (2023) (see Figure 1) that maps the mathematical processes students use when solving problems, illustrates how these processes interconnect with one another and with 21st-century skills, and emphasizes the application of mathematics in a variety of real-world contexts [15]. This model captures the multifaceted nature of ML through its interrelated components, including mathematical reasoning along with its three constituent processes, content knowledge, contexts, and 21st-century skills.

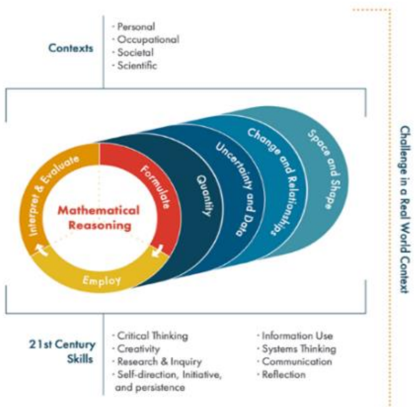

Figure 1. ML framework [28]

2.1.2. RME for mathematical literacy

ML, by definition, demands that PISA items be genuinely authentic, presenting students with problems that mirror how mathematics is applied in real-world situations outside of school [29]. In RME theory, context serves as the essential starting point for students to investigate mathematical concepts within situations that feel “experientially real” to them [30, p. 111]. To address problems set in realistic contexts, the RME approach emphasizes developing students’ problem-solving abilities by relating mathematical concepts to real-world situations, fostering direct engagement in the problem-solving process, and stimulating critical thinking [31]. Students are encouraged to "reinvent" mathematics [32, p. 3] rather than passively receiving information, which includes developing formal mathematics from informal understanding. According to Van den Heuvel-Panhuizen & Drijvers (2020), this instructional process is guided by six core RME principles: activity, reality, level, intertwinement, interactivity, and guidance [33]. The primary goal, as noted by Antasari et al. (2023, as cited in Yusmi et al., 2024), is to shift students’ perceptions of the subject from difficulty to accessibility, thereby fostering a deeper and more meaningful learning experience [34].

Compared to deductive approaches, RME leads to significantly greater improvements in ML. For instance, a study conducted by Yusmi et al. (2024) showed that students taught with RME reached an average ML level 4, whereas those taught with a deductive approach only reached an average level 2 [34]. The study by Çakıroğlu et al. (2024) also revealed that RME improves all dimensions of ML: formulating, employing, and interpreting and evaluating, and while improvements were seen across the board, the “interpret” dimension sometimes showed the lowest gain, highlighting the importance of discussion [14]. In RME, vertical mathematization occurs when a student adopts a more sophisticated, organized, or “more mathematical” solution-a shift that discussions of diverse methods and insights readily foster [35, p. 783].

2.1.3. DAPIC problem-solving process for mathematical literacy

The DAPIC problem-solving cycle-an acronym for Define, Assess, Plan, Implement, and Communicate-represents a structured, five-stage methodology designed to guide individuals through complex problem resolution, particularly within educational contexts. As noted by Meier et al. (1996), although real-life problem solving can follow a linear D-A-P-I-C sequence, steps may be omitted, repeated, or executed in a different order [36]. They also emphasized that teachers should offer multiple entry points and varied opportunities for students to employ DAPIC in diverse ways, viewing it as a recursive process rather than a rigid sequence, as illustrated in Figure 2. The DAPIC cycle exhibits a direct and profound alignment with the PISA framework for ML, particularly concerning its core problem-solving dimensions.

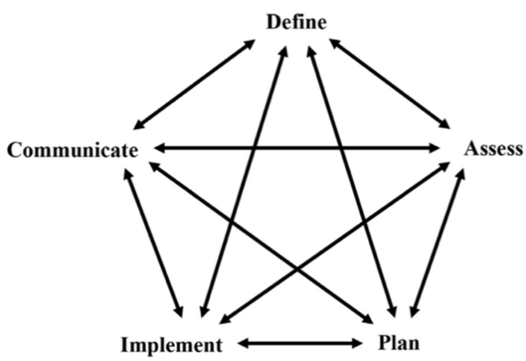

Figure 2. Interaction in the DAPIC problem-solving process [36, p. 236]

In the initial stages, the Define and Assess phases parallel the formulation process, where realistic problems are decoded and translated into mathematical models through reasoning. This leads to the Plan and Implement phases, which correspond to the employ process, where mathematical tools and strategies are applied to derive solutions. Finally, the Communicate phase aligns with the interpret and evaluate process, requiring students to critically examine results within their original context and articulate the reasoning behind their conclusions to others.

Mathematical reasoning serves as the essential foundation for all these phases, from the initial selection of relevant information and variables to the logical computation and the critical examination of conclusions.

2.2. Instructional process for developing mathematical literacy

2.2.1. Framework of instructional process

Drawing on RME principles, the DAPIC problem-solving cycle, the concept of ML, and the research of Sumirattana et al. (2017) and Çakıroğlu et al. (2024) [13], [14], we developed the instructional process shown in Figure 3. By mapping theoretical principles directly onto classroom actions, the design illustrates how specific teaching steps lead to desired student outcomes. Ultimately, this progression is intended to develop ML and equip students with the ability to formulate, employ, and interpret solutions while cultivating core mathematical reasoning and 21st-century skills. As mentioned earlier, mathematical reasoning underlies every stage of the DAPIC cycle. Hence, although the five-step instructional cycle is maintained, its scope is extended: beyond the original three dimensions, it also deliberately fosters mathematical reasoning. Furthermore, we emphasize that each phase of the instructional process simultaneously promotes a range of 21st-century skills in students. The instructional process unfolds through the following steps:

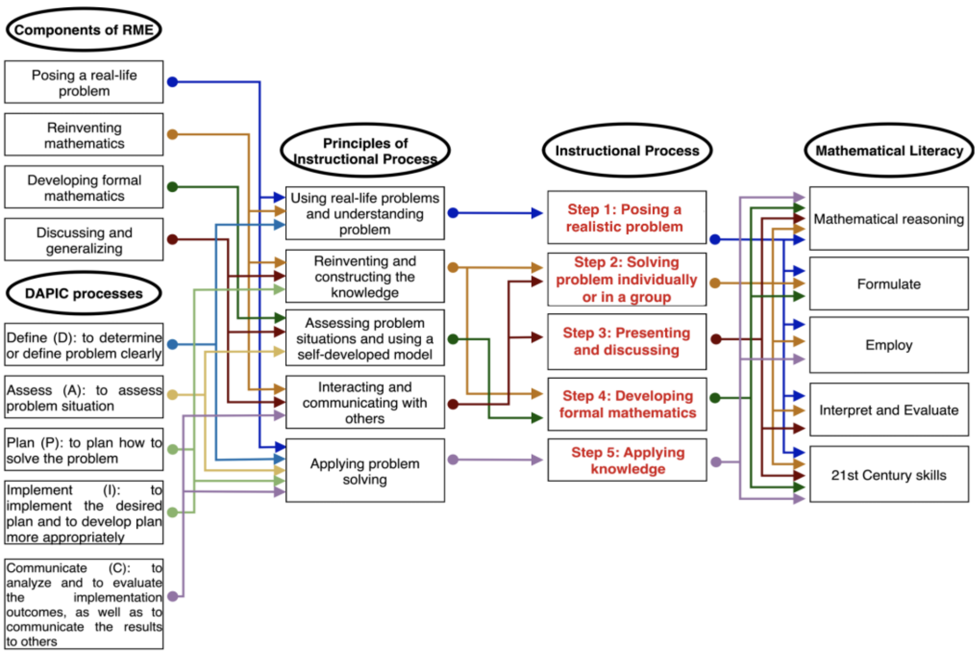

Figure 3. Framework of the instructional process (Prepared by authors)

Step 1: Posing a realistic problem. This step centers on the teacher presenting realistic problems tied to mathematical concepts, which offer multiple solution paths and prompt students to analyze and define the problem [13]. In RME, problem solving is not about following fixed procedures in predetermined situations but about exploring diverse ways to arrive at a solution [37]. The connection to reality in RME functions not only as the domain for applying mathematical skills at the end of instruction but also as the fundamental source from which students construct mathematical knowledge [37]. Connecting lessons to realistic scenarios fosters inquiry and critical thinking as students identify key details, while choosing how to approach the task builds self-direction and persistence.

Step 2: Solving problems individually or in a group. Focusing on gathering problem-related data and assessing the situation to formulate a solution plan, this step involves developing an accessible, student-generated model for individual or group work. In this phase, teachers act as facilitators who encourage diverse strategies and heuristics while guiding students through conceptual difficulties [13]. Students are motivated to exchange their approaches and offer feedback on each other's strategies [14]. Whether working individually or collaboratively, students exercise creativity in crafting solutions, information literacy in selecting relevant data, and, within teams, communication and systems thinking to integrate diverse ideas.

Step 3: Presenting and discussing. This step centers on presenting and discussing solutions-examining various problem-solving methods for correctness, adequacy, and efficiency; interpreting the problematic situations, and encouraging students to compare and justify their approaches with peers [13]. Furthermore, incorporating digital tools during this stage enables students to venture into novel areas, forecast outcomes, and interpret their results [38]. Sharing and critiquing solutions sharpens communication and critical thinking, sparks creativity through exposure to alternative perspectives, and fosters reflection on the most effective strategies.

Step 4: Developing formal mathematics. This step focuses on solving similar problems and discussing methods to develop solution procedures, involving dialogue between students and teachers to verify and advance both conceptual and procedural mathematical understanding [13]. Students' informal strategies frequently precede formal procedures, and mathematizing these similar solution methods embodies the guided reinvention process [35]. Translating informal models into standard ones fosters systems thinking, demands critical thinking to justify steps, and encourages creativity in exploring multiple formal approaches.

Step 5: Applying knowledge. This step emphasizes applying the developed conceptual and procedural mathematical knowledge to address a range of problems, including realistic contexts [13]. Adapting methods to new contexts promotes research and inquiry, requires critical thinking to interpret results, strengthens systems thinking by testing model limits, and fosters self-direction as students refine their understanding.

In summary, these instructional steps are designed to address all components of ML, especially mathematical reasoning, which lies at the heart of the problem-solving process. Step 1 presents realistic, context-rich problems that require students to deconstruct situations, identify relevant quantities and assumptions, and translate the context into a mathematical model. These practices strengthen formulation skills and reasoning from real data. In step 2, students develop and test their own models (individually or collaboratively), detect patterns, refine conjectures, and evaluate various approaches, thereby exercising inductive and strategic reasoning while learning to justify intermediate steps. Step 3 requires learners to present and critique solutions, which renders thinking explicit, prompts the defense or revision of arguments, and fosters evaluative and justificatory reasoning. Step 4 transitions informal ideas into formal notation and procedures, supporting abstraction, generalization, and the construction of deductive chains of reasoning that explain the mechanics of why methods work. Finally, step 5 requires applying methods to new contexts and testing model limits-activities that build reflective, evaluative, and metacognitive dimensions of mathematical reasoning.

2.2.2. Example

The following example illustrates each step of the instructional process in a Grade 9 lesson on solving systems of linear equations in two variables. The instructional activities below are aligned with the recommendations of S Sumirattana et al. (2017) [13].

Step 1: Posing a realistic problem. The teacher formulates and presents a realistic problem to review prior knowledge of two-variable linear equations, establishing the foundation for learning techniques for solving the system of two equations. Using visual aids, such as pictures, stories, diagrams, or symbols familiar to students, is recommended (see Figure 4).

Problem 1: *A group of Vietnamese customers visited Starbucks and bought six drinks in total, including cold brew and caffe latte. Each cold brew costs 65,000 VND, while each caffe latte costs 60,000 VND, and the group paid a total of 380,000 VND.*

Figure 4. Illustration for problem 1 (Prepared by authors)

Figure 5. Intersection of two lines (Prepared by authors)

The teacher then poses the question, "How many drinks of each type did the group buy?" linking it directly to the lesson's objective: solving systems of linear equations in two variables. Students can deepen their understanding and clarify the problem by underlining or highlighting key information, such as "six drinks," "cold brew – 65 000 VND," "caffe latte – 60 000 VND," and "total = 380 000 VND". Alternatively, they may paraphrase the situation, for example: "A group buys six beverages for a total of 380 000 VND. Some are cold brews at 65 000 VND each, and the rest are lattes at 60 000 VND each."

Step 2: Solving problems individually or in a group. Students now gather problem-related information by defining variables, such as $\boldsymbol{x}$ for the number of cold brews and $\boldsymbol{y}$ for the number of caffe lattes, and identifying the constraints: the total number of drinks ($x + y = 6$) and the total cost ($65\ 000x + 60\ 000y = 380\ 000$). They then analyze the situation and plan their approach by formulating the question as: "Find the values of $\boldsymbol{x}$ and $\boldsymbol{y}$ that satisfy both the quantity and cost conditions," effectively transforming a realistic scenario into a formal mathematical problem. Next, students develop their own models to solve the problem by drawing on their prior knowledge or familiar strategies, working either individually or in groups. These student-led approaches may include constructing a table for trial and error (see Table 1), graphing the equations as lines and finding their intersection point (see Figure 5), or applying informal techniques, such as "substitution" or "elimination". It is noteworthy that although students have not yet been taught the formal terminology in this phase, they are capable of devising and applying these strategies intuitively.

Table 1. Table for the trial and error method

Trial	**Number of Cold Brews (65 000 VND/cup)**	**Number of Caffe Lattes (60 000 VND/cup)**	**Number of Cups (Target: 6)**	**Total price (Target: 380 000 VND)**	**Check**
1	3	3	6	375 000	Lower
2	5	1	6	385 000	Higher
3	*4*	*2*	*6*	*380 000*	*Correct*

During this phase, the teacher supports students by modeling strategies and heuristics, such as the "thinking-aloud" technique – solving the problem on the board while verbalizing each decision (e.g., "If I draw a table, I can see how different values pair to give the total cost."). This approach externalizes invisible reasoning, making it accessible to all learners. The teacher provides differentiated individual or group guidance as needed.

Step 3: Presenting and discussing. The teacher invites students or groups-ideally organized by the chosen strategies (table, graphing, substitution, elimination)-to explain their methodology to the class. For example:

- The "table" group can walk through how they set up rows of x and y values to find a match;
- The "graphing" group can sketch their lines on the board or use software like GeoGebra to show the intersection.
- The "substitution" group can illustrate how they isolated one variable to solve the system.
- The "elimination" group can show how they added or subtracted equations to cancel out a variable.

The teacher should encourage presenters to articulate why they chose a specific method, demonstrate each procedural step (e.g., plotting axes, isolating variables, or aligning coefficients), and finally validate their solution against the original problem constraints.

Once all methods have been presented, the teacher facilitates a class discussion that evaluates correctness, clarity, and efficiency. By asking targeted questions, the teacher helps students recognize which approaches yielded an exact solution most efficiently—often pointing toward elimination—and how the table method might reveal patterns that algebraic methods obscure. The discussion further explores how graphing visualizes the relationship between equations despite certain limitations, and in what scenarios substitution feels more intuitive or error-prone than other techniques. To synthesize these insights, a graphic organizer such as a T-chart is used to capture the pros and cons of each strategy. The teacher then prompts students to critique their own solutions by paraphrasing a classmate's method, defending their own chosen approach, and judging which strategies generalize best to complex systems or non-integer solutions. This Think-Pair-Share structure ensures that every student contributes to assessing the viability and efficiency of all student-developed methods.

Figure 6. Illustration for problem 2 (Prepared by authors)

Step 4: Developing formal mathematics. The teacher presents another realistic problem that students can solve individually or collectively using the same problem-solving methods:

Problem 2: *A family plans to travel by car from Hanoi to visit Ta Xua (Son La province) this summer. If the car travels at 40 km/h, they arrive at Ta Xua 90 minutes later than planned. However, if it travels at 60 km/h, they arrive 30 minutes earlier than planned. Determine the distance and the originally planned travel time (see Figure 6).*

Students can formulate the system of linear equations: $x - 40y = 60$ and $-x + 60y = 30$, where x denotes the distance (in kilometers) from Hanoi to Ta Xua and y the planned travel time (in hours). They might realize that using a table with trial-and-error or a hand-drawn graph becomes cumbersome here because the true solution for x (distance) and y (time) does not land on "nice" whole-number pairs, which means that guessing via a table would require testing many fractional values to get the total exactly right - an extremely time-consuming process. Moreover, plotting the lines demands a large, accurately scaled coordinate grid. Even small drawing or measurement errors can throw the intersection point, and thus the precise distance and time, far off. Therefore, students may turn to alternative algebraic methods.

The teacher now introduces two algebraic techniques—substitution and elimination—that students may have informally used in the previous problem but are now presented as formal problem-solving methods. Through guided discussions with the teacher, students refine their conceptual and procedural understanding by adopting formal mathematical language. For instance, if students choose the elimination method, the teacher might ask: "Why is this technique useful?"; "Which variable are we aiming to eliminate?"; "How did that variable cancel out?"; or "What does the resulting equation reveal?" Alternatively, if using substitution, they could solve the first equation for x, then substitute into the second. The teacher guides students to explain each algebraic step using precise mathematical language: "We substitute the expression for x into the second equation, yielding a single-variable equation in y."

Together, the teacher and students now collaboratively distill the steps into a procedural framework. Although both substitution and elimination can be used to solve the system, the teacher prompts students to reflect with questions like: "Which method occurred to me first?"; "How confident am I in isolating variables?"; "Am I comfortable multiplying both sides of an equation by a constant?"; "Can I easily add or subtract linear equations?"; or "Do I feel confident finding least common multiples to set up elimination?" Their responses empower them to decide which method is most effective for their individual reasoning style.

Step 5: Applying knowledge. In this final stage, the teacher assigns several problems, including realistic scenarios, where students apply their developed conceptual and procedural mathematical knowledge.

Problem 3: *Two workers together can build a wall in 3 hours and 45 minutes. However, after working together for 3 hours, the first worker was assigned to another task, and the second worker continued alone for 2 more hours to finish the job. How long would it take each worker to complete the wall if working alone (see Figure 7)?*

Problem 4: *A company planned to produce 1, 000 boxes of face masks according to its initial schedule. However, due to the outbreak of the COVID-19 pandemic, they accelerated production by 10 boxes per day to meet market demand. ASA result, the production was completed 5 days ahead of schedule. How many days was the company originally planning to spend on production (see Figure 8)?*

Figure 7. Illustration for problem 3 (Prepared by authors)

Figure 8. Illustration for problem 4 (Prepared by authors)

The problems introduced to students should be arranged in order of increasing difficulty and complexity. Problem 1 is the most straightforward because the variables represent discrete physical counts and the relationships are simple sums of quantities and costs. While Problem 2 requires more careful unit conversions, it still relies on the familiar formula relating distance, velocity, and time. In contrast, Problems 3 and 4 demand more abstract modeling. The variables in Problem 3 must represent the work rate (the fraction of work done per unit of time), where the "total work" is typically represented by the constant 1. This setup in Problem 4 involves a change in the daily rate that impacts the overall duration, leading to a more complex system that requires sophisticated algebraic manipulation to solve.

Students now analyze each problem and thoughtfully apply the developed procedures most appropriate to solving it. The teacher offers guidance and support as needed by using targeted prompts to scaffold students' thinking without doing the work for them. For example, the teacher might prompt problem-type recognition by asking, "Does this remind you of situations where work is shared, or where rates change over time?" or "Which details correspond to rates, time, and total output?" To help with modeling, the teacher could require, "What should x and y represent here?" or "How would you express the work done per hour?" If a student struggles to formulate the equation in problem 4, the teacher might pose, "When the rate increases by 10 and the time decreases by 5 days, what quantity remains constant?"

Throughout, the teacher provides light scaffolding, such as clarifying the equations or offering initial steps like isolating a variable, while allowing students to execute the reasoning and solution process themselves. During this process, students take ownership of analyzing realistic problem structures and choosing appropriate algebraic strategies, while the teacher plays a responsive and supportive role, which fosters both mathematical independence and strategic thinking. Through targeted guidance, mathematical reasoning can be actively developed. Ultimately, this student-centered approach cultivates essential 21st-century skills, including critical thinking and self-direction, as students practice adapting mathematical knowledge to more complex scenarios.

3. Conclusions

To conclude, our instructional model weaves RME principles with the DAPIC problem-solving cycle to develop full-spectrum ML in Grade 9 students. By centering lessons on context-rich problems and guiding learners through defining, assessing, planning, implementing, and communicating solutions, we align every phase with PISA's core processes - reasoning, formulating, employing, interpreting, and evaluating - while embedding critical 21st-century skills. Classroom examples on linear equation systems demonstrate how students transition from informal strategies to formal methods and deepen both conceptual understanding and procedural fluency. Teacher prompts and collaborative discussion further foster students' metacognition and adaptability. As suggested by Sumirattana et al. (2017), teachers should analyze students' backgrounds to select problems that enhance their understanding and problem-solving capabilities, then patiently allow them to develop their own procedures while offering guided heuristics as needed [13]. Moving forward, targeted research could measure this approach's impact across diverse settings, and professional development should equip educators to design realistic tasks and facilitate dynamic inquiry.

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